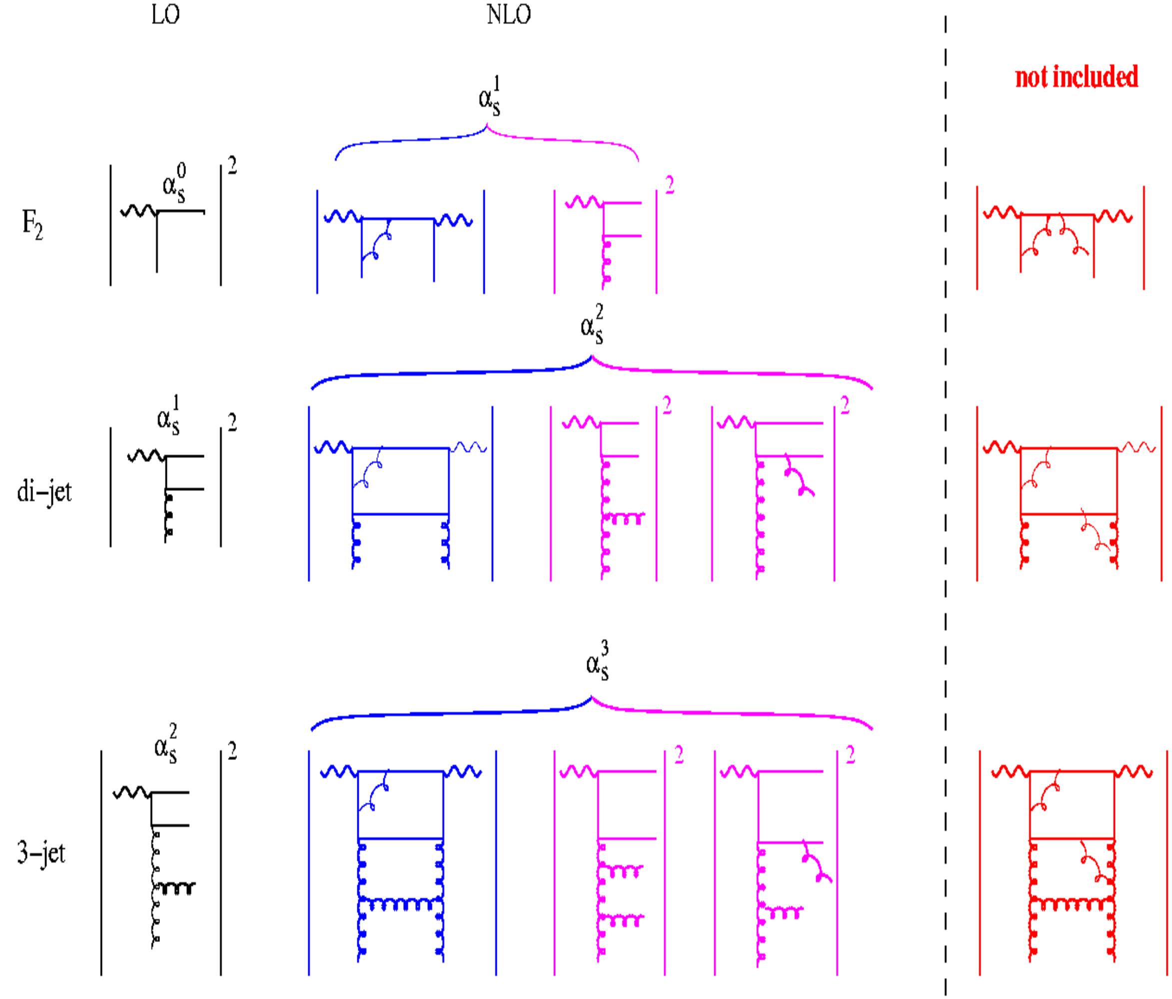
QCD and Collider Physics: Heavy Quarks, Fragmentation-Hadronization

- Resume from last lecture
- Heavy Quarks: fragmentation functions, massive/massless approach
- Dijet production in DIS (in LO and NLO)
- Approaches to even higher orders
  - Parton showers
  - unintegrated pdfs
- Fragmentation/hadronization

http://www-h1.desy.de/~jung/qcd_collider_physics_2005

From LO to NLO ...

- NLO for F_2: $O(\alpha_s)$
- NLO for dijets: $O(\alpha_s^{\ 2})$
- NLO for 3-jets: $O(\alpha_s^{\ 3})$



NOTE: NLO for dijets is **NOT** NNLO for F_2

NLO calculations: principles

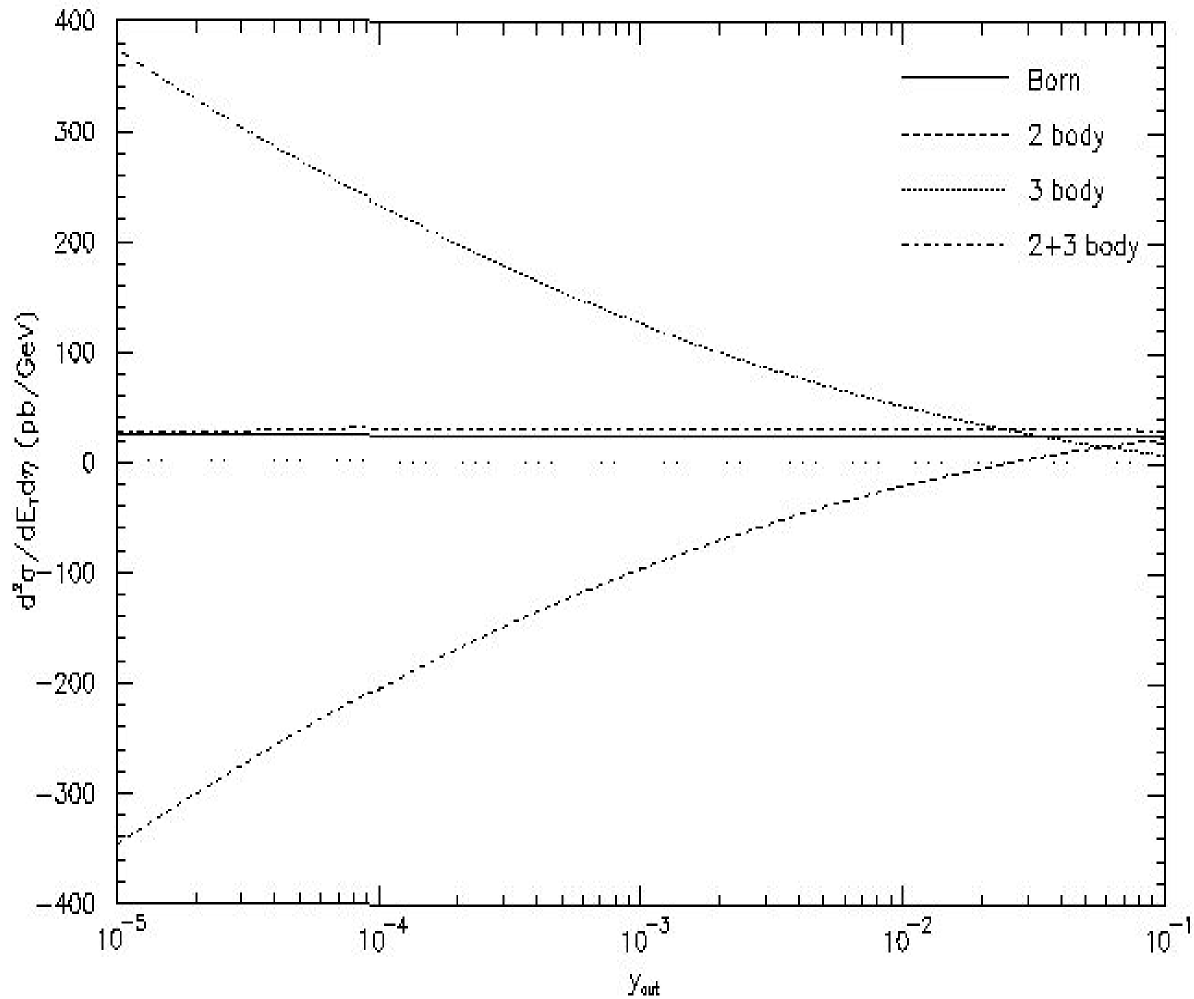
$$\sigma = \int_m d\sigma^{\mathrm{Born}} + \int_m d\sigma^{\mathrm{Virtual}} + \int_{m+1} d\sigma^{\mathrm{Real}}$$

- Virtual (1-loop) corrections:
  - UV
  - IR
  - collinear
- UV corrections handled by renormalization procedure
- soft/collinear singularities do not cancel within $d\sigma^V$
  - only with appropriate quantities from $d\sigma^R$
- cancellation is guaranteed by: $F^{m+1} \to F^m$
- both $d\sigma^V$ and $d\sigma^R$ are separately divergent.... need for regularization
  - massive gluon scheme
  - dimensional regularization
- Computation very difficult:
  - use hybrid of analytical and numerical methods
    - Phase space slicing method
    - Subtraction method

Phase Space Slicing

Klasen, Kleinwort, Kramer hep-ph/9712256

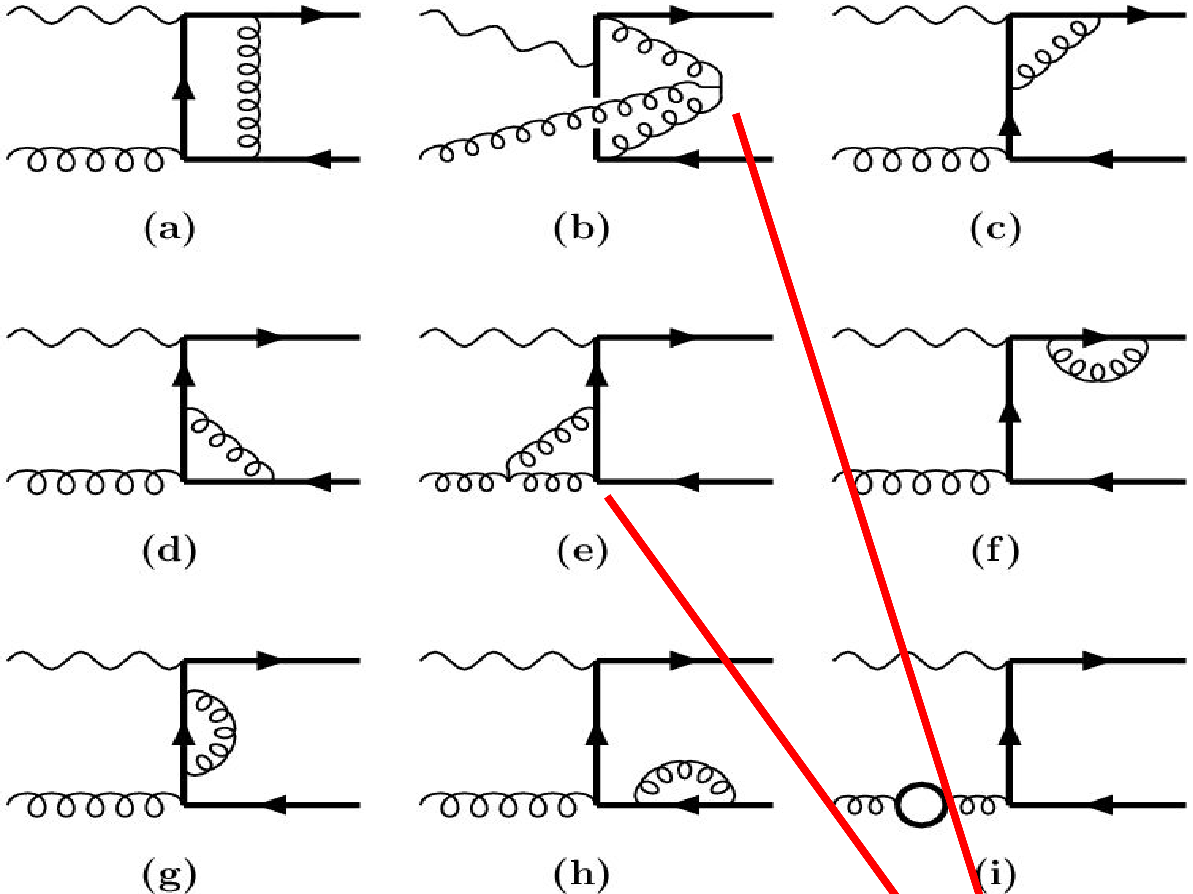
- define parameter y_{cut} to separate soft + virtual from finite real emissions.
- each contribution shows sensitivity
- but sum of all contributions is independent of y_{cut}



Heavy Quark: NLO virtual corrs

- one-loop virtual corrections:

I. Bojak, M. Stratmann Nucl.Phys.B540:345-381,1999



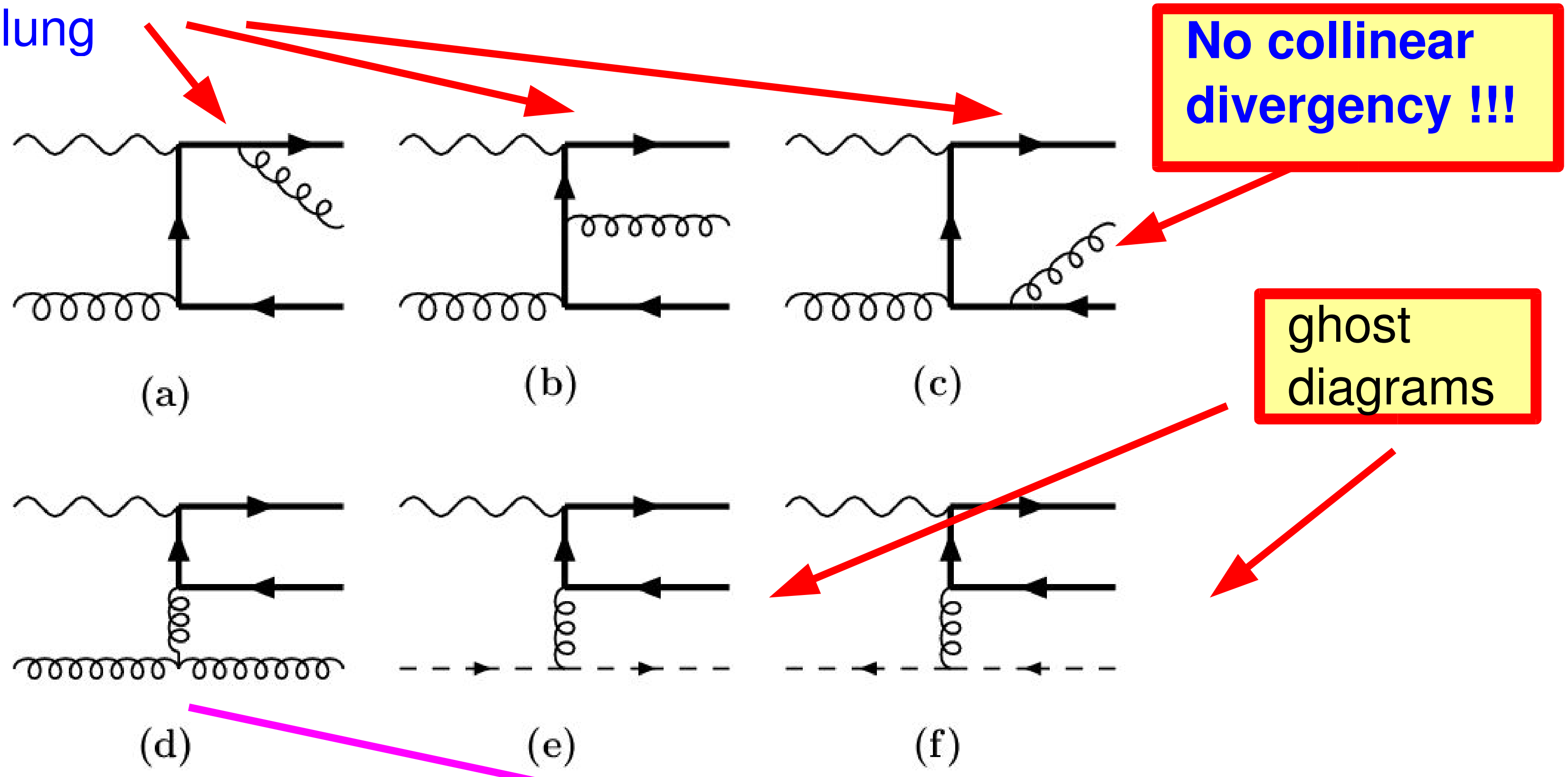
- at $\mathcal{O}(\alpha\alpha_s^2)$ only interference between virtual and born terms contribute

$$|\tilde{M}|^2_{VB} = 2\mathrm{Re}\left(\widetilde{M_V M_B^*}\right) = g_s^4 e^2 e_Q^2 \left[2C_F \tilde{V}_{QED} + C_A \tilde{V}_{\mathrm{non-abelian}}\right],$$

Heavy Quarks: NLO real corrections

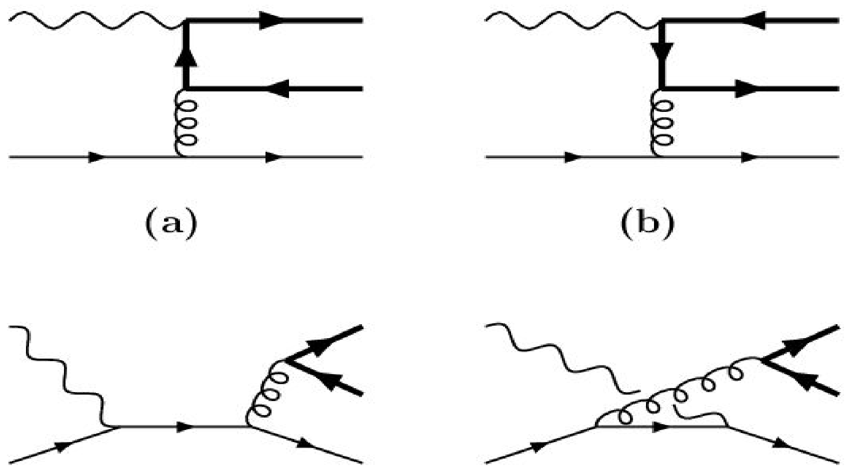
I. Bojak, M. Stratmann Nucl.Phys.B540:345-381,1999

- IR singularities of virtual x-sections are canceled by soft part of the gluon bremsstrahlung



$$|\tilde{M}_R|^2 = \widetilde{M_R M_R^*} = g_s^4 e^2 e_Q^2 \left[2C_F \tilde{R}_{QED} + C_A \tilde{R}_{\text{non-abelian}} \right]$$

Heavy Quarks: NLO quark corrs



Bethe Heitler graphs: A_1

Compton graphs: A_2

Interference: Bethe-Heitler-Compton: A_3

$$|\tilde{M_q}|^2 = \widetilde{M_q M_q^*} = g_s^4 e^2 \frac{C_F}{2}\left[e_Q^2 \tilde{A}_1 + e_q^2 \tilde{A}_2 + e_q e_Q \tilde{A}_3\right]$$

$$\frac{d^2\hat{\sigma}^{(1)}_{q\gamma}}{dt_1 du_1}(\mu_f^2) = \frac{d^2\tilde{\sigma}^{(1)}_{q\gamma}}{dt_1 du_1}(\mu^2) -$$

$$-\frac{\alpha_s}{2\pi}\int_0^1 dx_1 \left[\tilde{P}_{gq}(x_1)\frac{2}{\varepsilon} + \tilde{F}_{gq}(x_1,\mu_f^2,\mu^2)\right] x_1 \left[\frac{d^2\tilde{\sigma}^{(0)}_{g\gamma}}{dt_1 du_1}\right]\begin{pmatrix} s \to x_1 s \\ t_1 \to x_1 t_1\end{pmatrix} -$$

$$-\frac{\alpha}{2\pi}\int_0^1 dx_2 \left[\tilde{P}_{q\gamma}(x_2)\frac{2}{\varepsilon} + \tilde{F}_{q\gamma}(x_2,\mu_f^2,\mu^2)\right] x_2 \left[\frac{d^2\tilde{\sigma}^{(0)}_{q\bar{q}}}{dt_1 du_1}\right]\begin{pmatrix} s \to x_2 s \\ u_1 \to x_2 u_1\end{pmatrix}$$

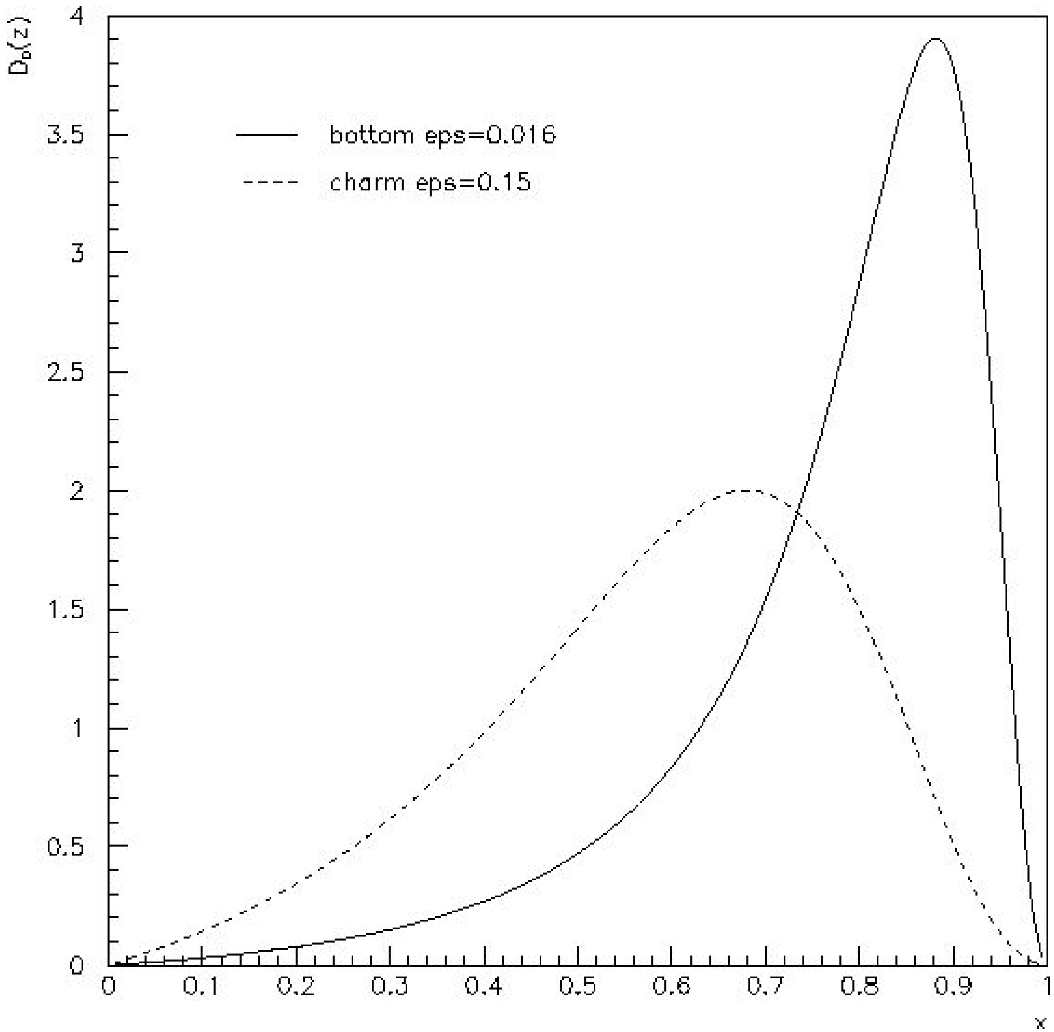
D* production

- transition from heavy quark to observable hadron by fragmentation function FF
- often Peterson FF is used:

$$D_Q(z) \sim \frac{1}{z}\left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right]^{-2}$$

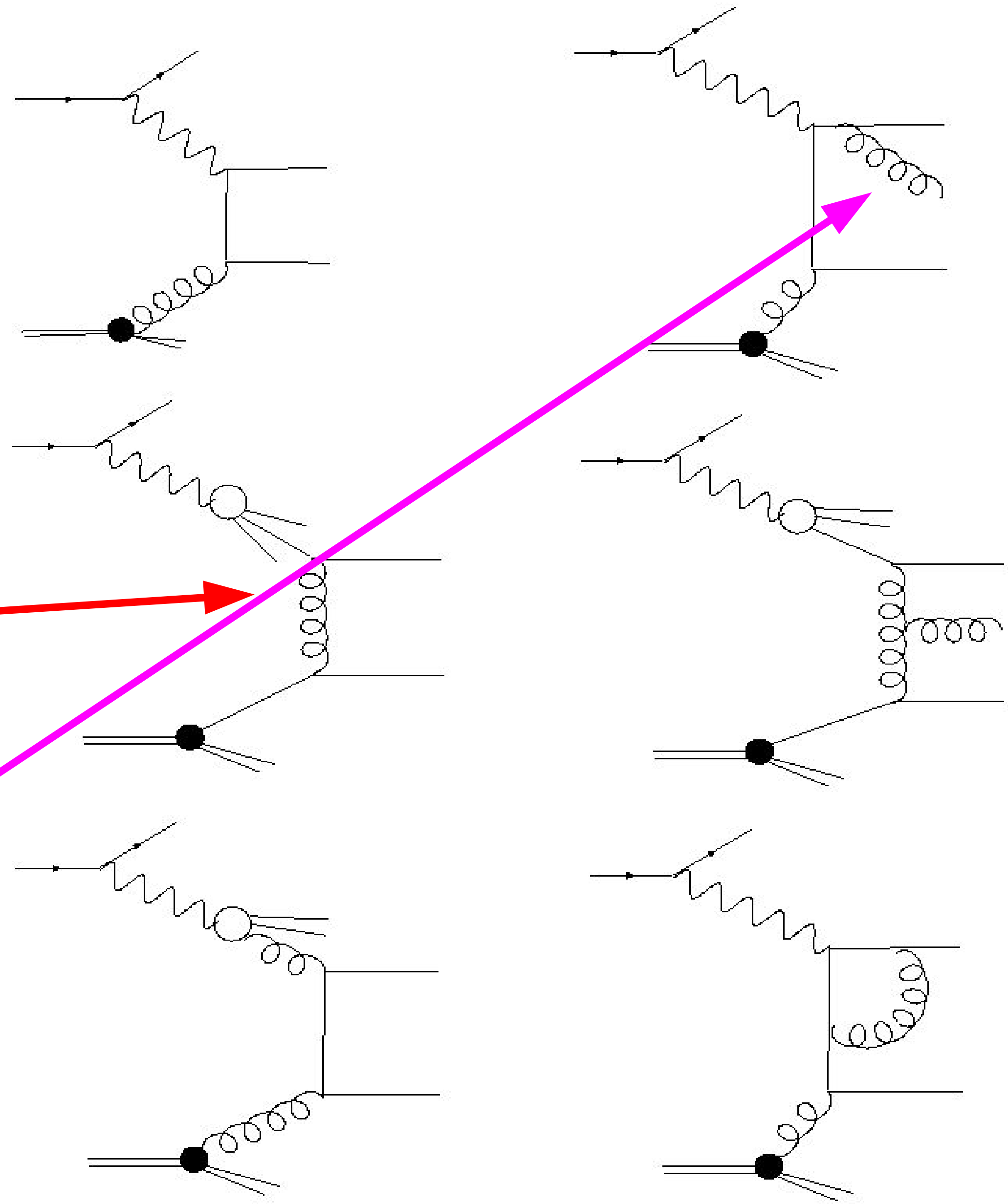
- In massive mode, no collinear divergencies ... nothing to be resummed ... apply fragmentation function directly to parton level calculation.
- Only if $p_t^2 \gg m^2$ large logs could appear:

$$\log\left(\frac{p_t^2}{m^2}\right)$$

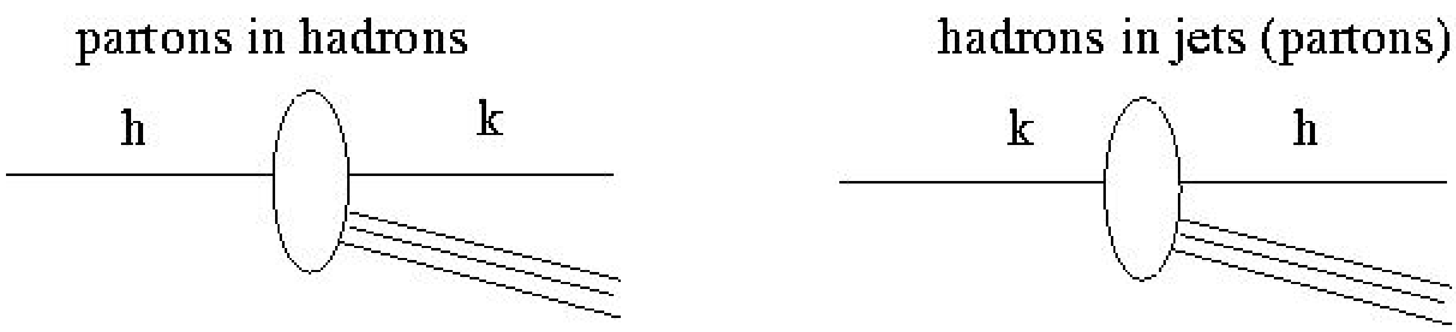


Heavy Quarks in NLO: massless

- all partons are treated massless
  - ➔ soft singularities cancel between real and virtual contributions
  - ➔ initial state collinear singularities are absorbed into PDFs
  - ➔ final state collinear singularities are absorbed into FF (not existing in massive case)
- some additional diagrams compared to massive case
- large logs appear here, which need to be resummed ... scale dependence of FF



Scaling violations of Frag. Fcts.



- Similarity with evolution of parton density functions

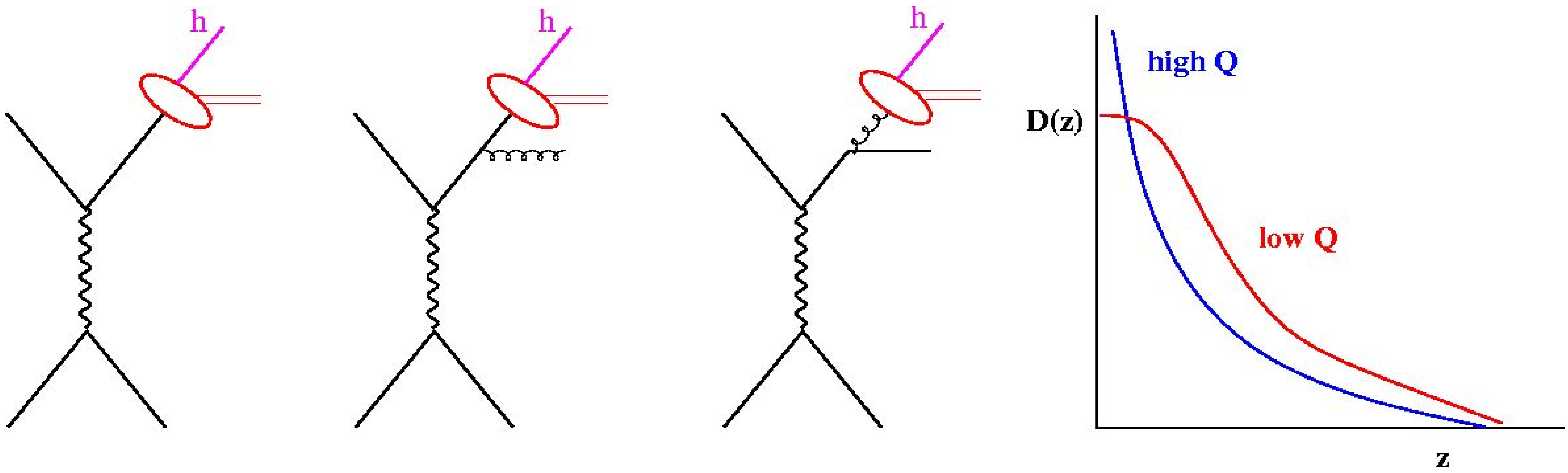
$$t\frac{\partial}{\partial t}D_i(x,t) = \sum_j \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{ji}(z,\alpha_s) D_j\left(\frac{x}{z},t\right)$$

- with splitting functions: $P_{ji}(x,\alpha_s) = P_{ji}^{(0)} + \frac{\alpha_s}{2\pi}P_{ji}^{(1)}$

- lowest order splitting functions are the same as in DIS case
- higher order $P_{gg},\ P_{qg}$ are more singular than in DIS

➔ resummation of small x enhanced terms have different behavior...

Fragmentation Functions



$$\frac{dD_q^h(z,\mu^2)}{d\log\frac{\mu^2}{\Lambda^2}} = \frac{\alpha_s}{2\pi}\int_0^1 \frac{dy}{y}\left(P_{qq}(y)D_q^h(z/y) + P_{gq}(y)D_g^h(z/y)\right)$$

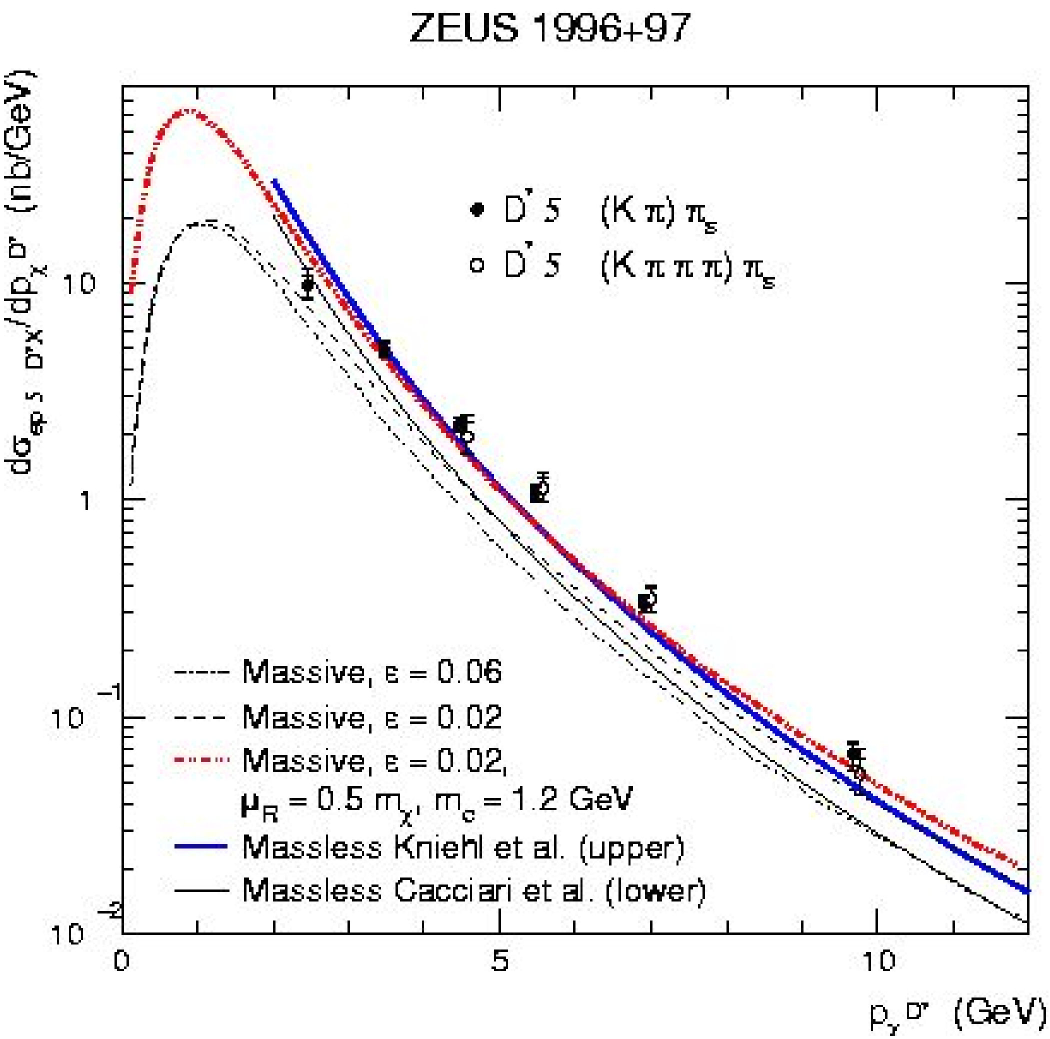
$$\frac{dD_g^h(z,\mu^2)}{d\log\frac{\mu^2}{\Lambda^2}} = \frac{\alpha_s}{2\pi}\int_0^1 \frac{dy}{y}\left(P_{qg}(y)\sum_i \left(D_q^h(z/y) + D_{\bar{q}}^h(z/y)\right) + P_{gg}(y)D_g^h(z/y)\right)$$

D production*

- transition from heavy quark to observable hadron by fragmentation function FF
- often Peterson FF is used:

$$D_Q(z) \sim \frac{1}{z}\left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right]^{-2}$$

- apply FF to parton level calculation for comparison with measurement
- watch out for different FF in massive and massless approach !



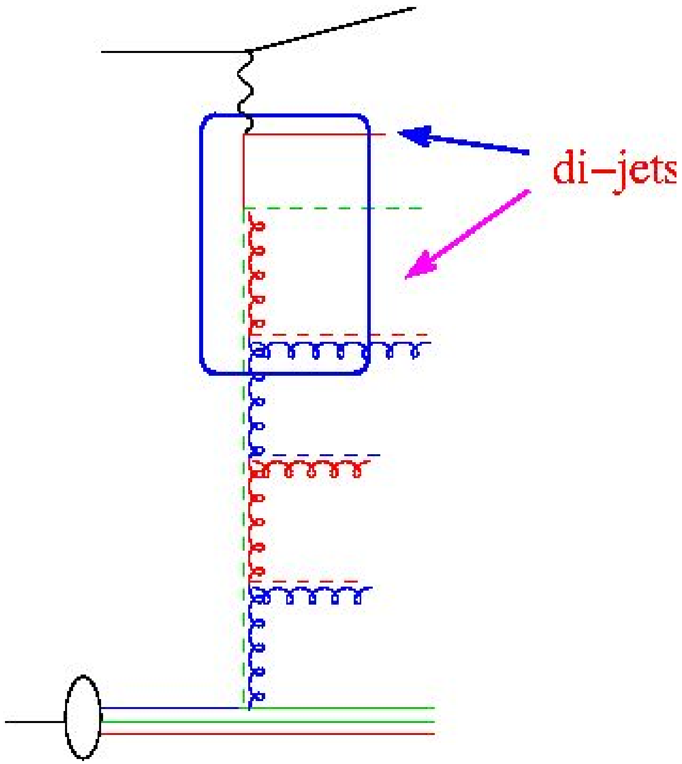
Heavy Quark production: jets

S. Frixione, G. Ridolfi Nucl.Phys.B507:315-333,1997

- ensure proper cancellation of real and virtual corrections
- matching of

$$2 \rightarrow 3 \;\; to \;\; 2 \rightarrow 2$$

  - apply asymmetric pt-cuts for jets



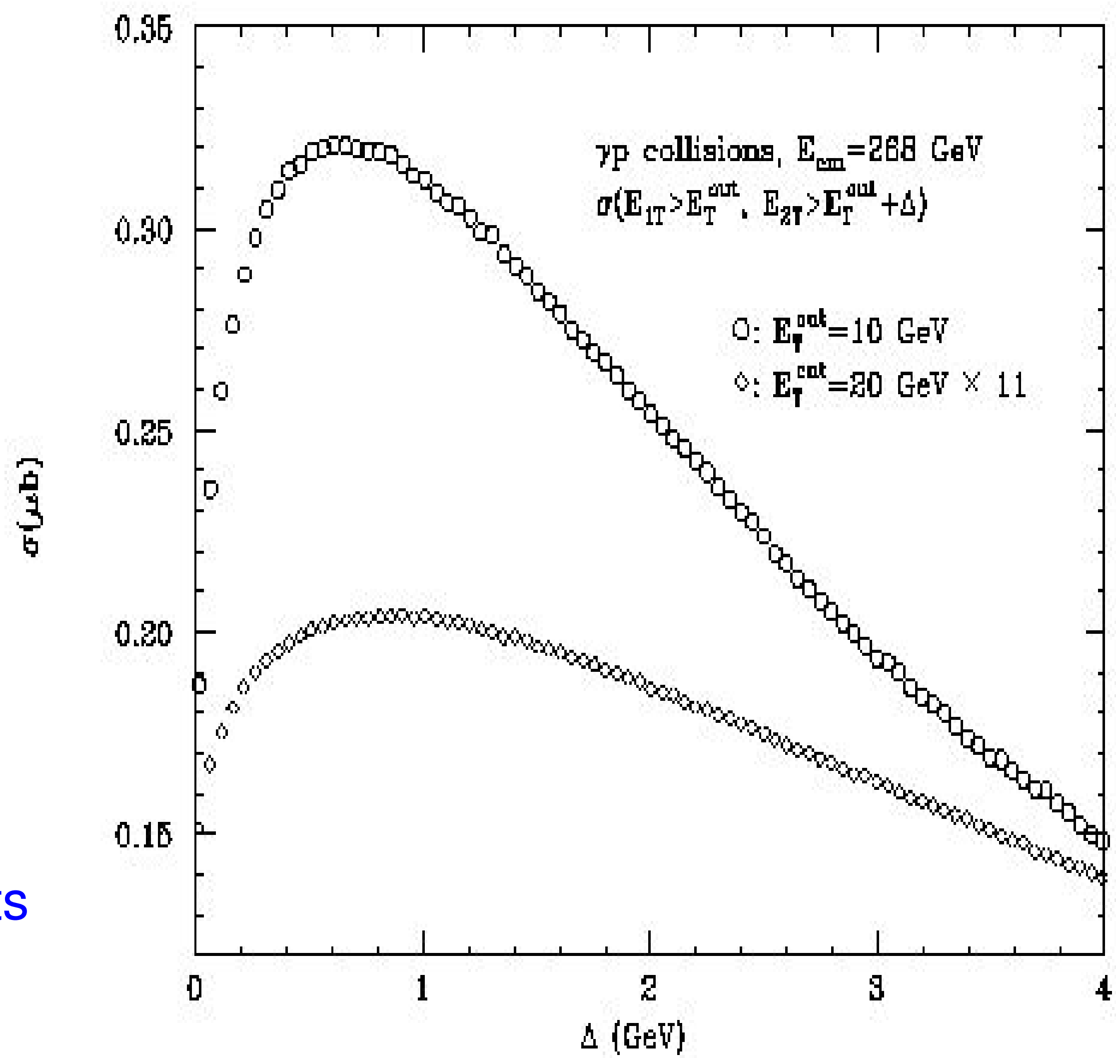
Heavy Quark production

S. Frixione, G. Ridolfi Nucl.Phys.B507:315-333,1997

- ensure proper cancellation of real and virtual corrections
- matching of

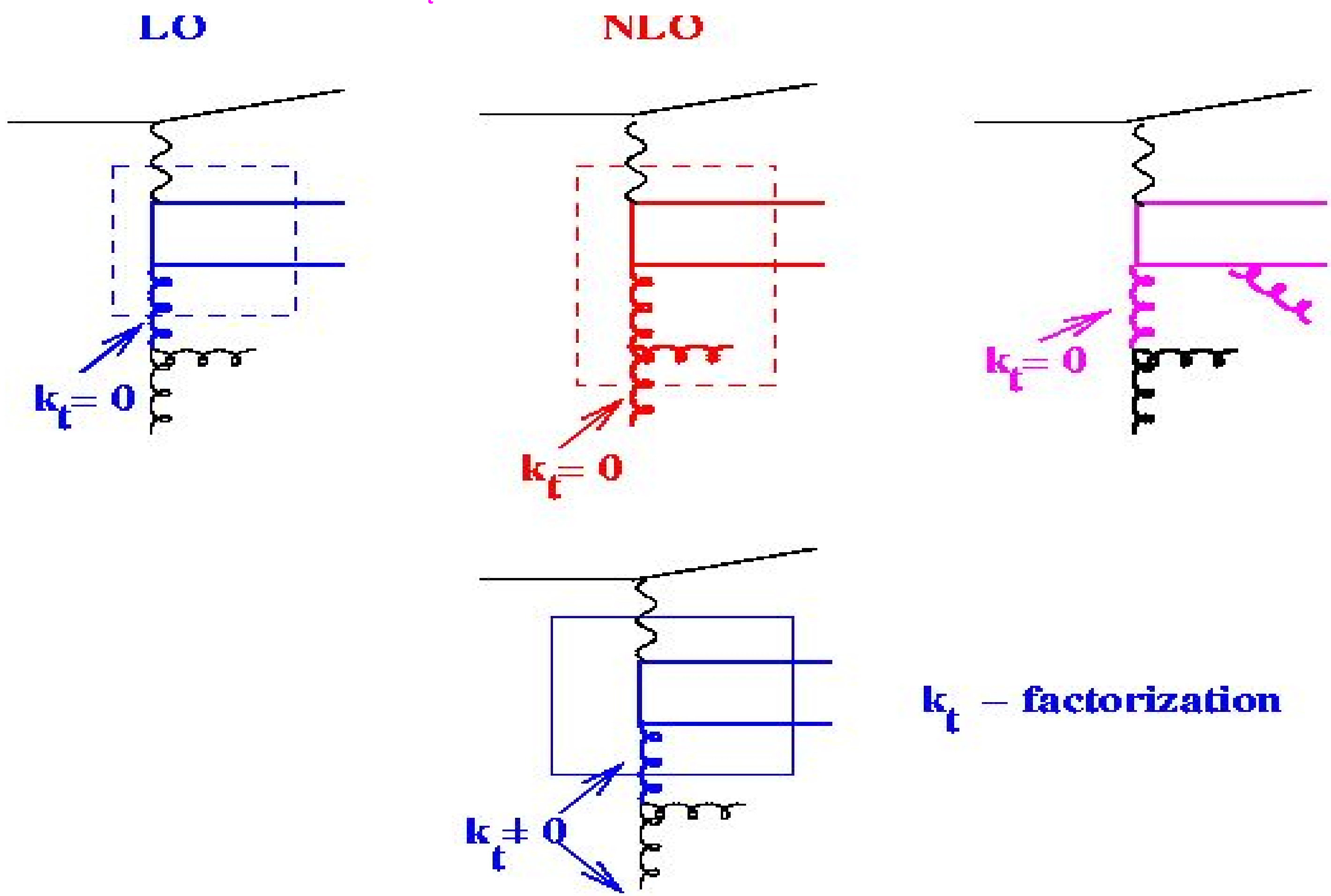
$$2 \rightarrow 3 \;\; to \;\; 2 \rightarrow 2$$

  - apply asymmetric pt-cuts for jets
  - average of transverse momenta of jets
  - or....
- stay as inclusive as possible...
  - define infrared safe observables !
- recently significant improvements by resummation of soft gluons



k_t-factorization and collinear NLO

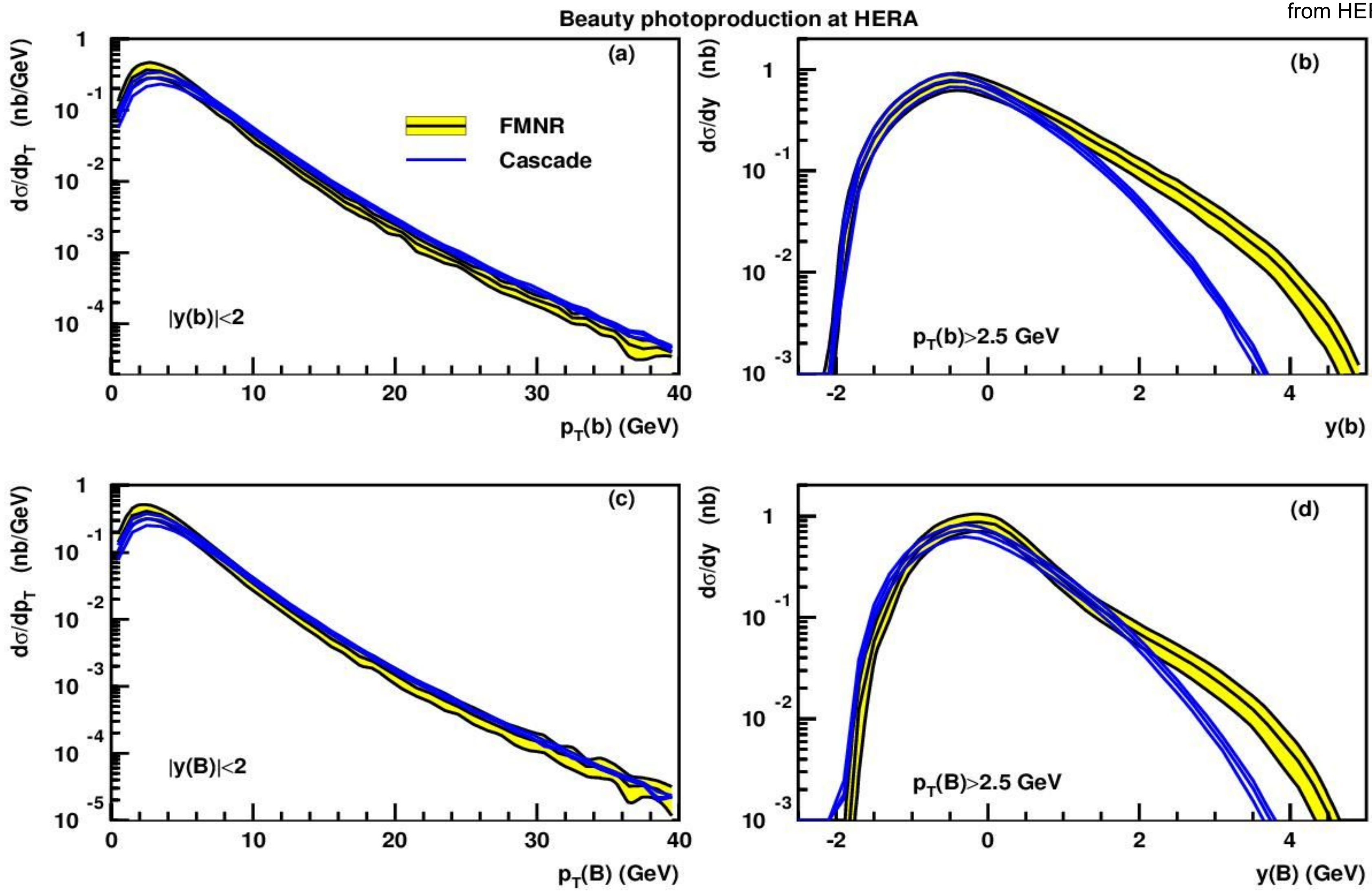
- off-shell matrix elements (k_t – factorization) includes part of NLO corrections:



- even soft k_t region is properly treated (not the case in part.level NLO calc)
- in addition contributions to all orders are included

Beauty at HERA

from HERA-LHC WS



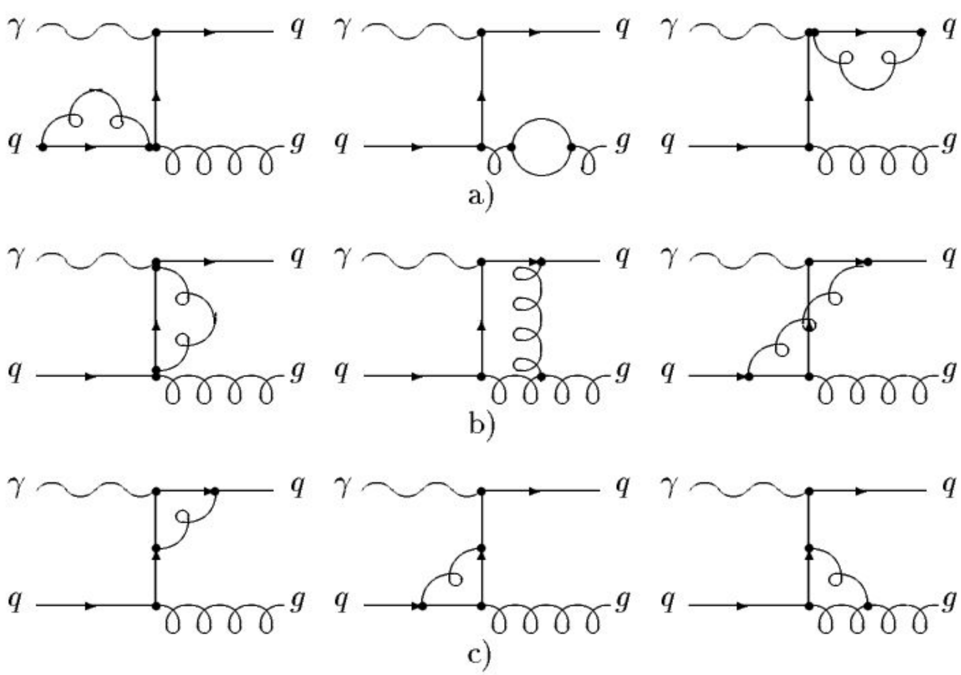
"Perfect" agreement of NLO(FMNR) calc with
CASCADE on quark and hadron level for $x<0.01$

!!!

Jets in NLO: quark induced

Klasen, Kleinwort, Kramer hep-ph/9712256

- DIS: virtual corrs for QCDC
  - real emissions
  - also diagrams for BGF
- photo production:
  - even more diagrams contribute:
  - resolved photons ...



Cancellation of individual contributions

Klasen, Kleinwort, Kramer hep-ph/9712256

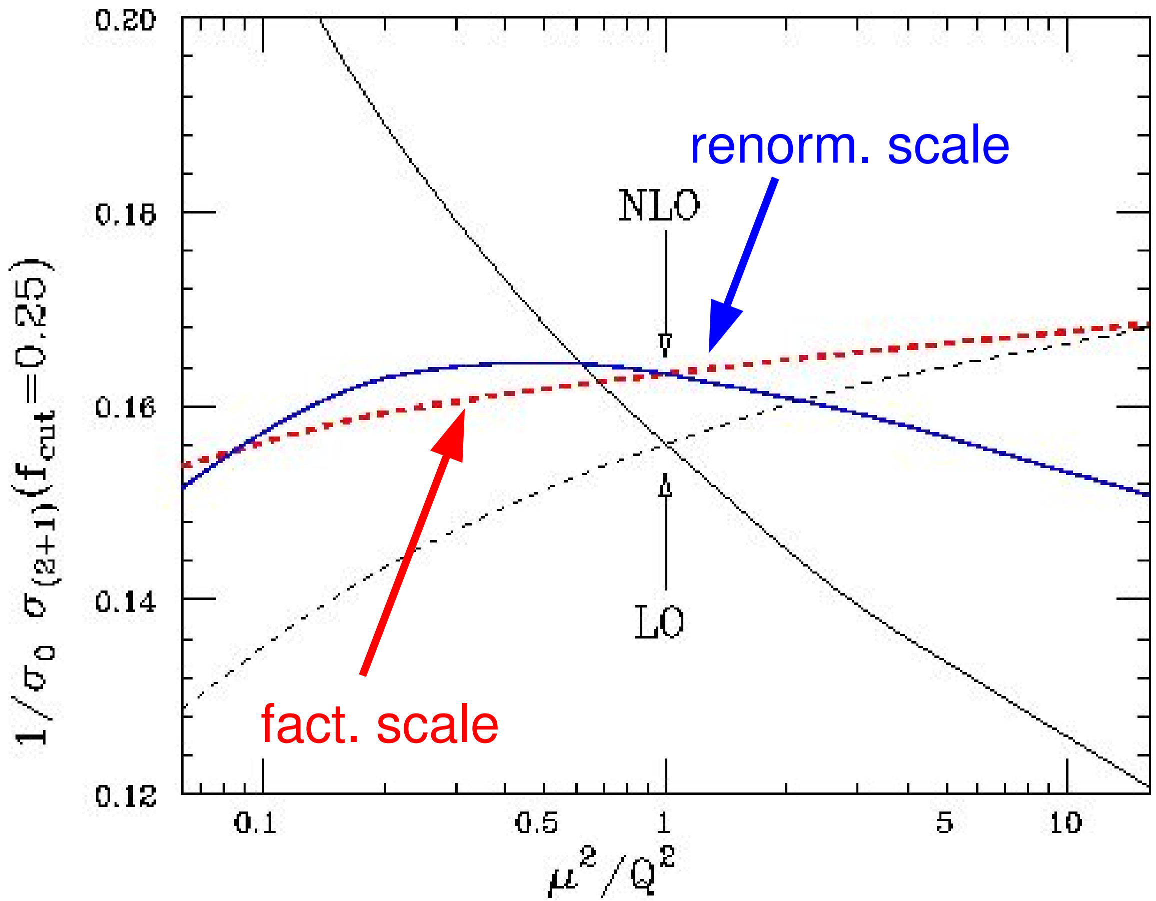
Process	Color Factor	NLO Correction	Singular Parts of Matrix Elements
$\gamma q \to gq$	C_F	Virtual Corr.	$\left[-\frac{2}{\varepsilon^2}-\frac{1}{\varepsilon}(3-2l(t))\right] T_{\gamma q\to gq}(s,t,u)$
		Final State	$\left[+\frac{1}{\varepsilon^2}+\frac{1}{2\varepsilon}(3-2l(t))\right] T_{\gamma q\to gq}(s,t,u)$
		Initial State	$\left[+\frac{1}{\varepsilon^2}+\frac{1}{2\varepsilon}(3-2l(t))\right] T_{\gamma q\to gq}(s,t,u)$
	N_C	Virtual Corr.	$\left[-\frac{1}{\varepsilon^2}-\frac{1}{2\varepsilon}\left(\frac{11}{3}-2l(s)+2l(t)-2l(u)\right)\right] T_{\gamma q\to gq}(s,t,u)$
		Final State	$\left[+\frac{1}{\varepsilon^2}+\frac{1}{2\varepsilon}\left(\frac{11}{3}- \quad l(s)+ \quad l(t)- \quad l(u)\right)\right] T_{\gamma q\to gq}(s,t,u)$
		Initial State	$\left[\quad +\frac{1}{2\varepsilon}(\quad - \quad l(s)+ \quad l(t)- \quad l(u))\right] T_{\gamma q\to gq}(s,t,u)$
	N_f	Virtual Corr.	$+\frac{1}{3\varepsilon}T_{\gamma q\to gq}(s,t,u)$
		Final State	$-\frac{1}{3\varepsilon}T_{\gamma q\to gq}(s,t,u)$

Table 7: *Cancellation of IR singularities from virtual, final state, and initial state NLO corrections for the direct partonic subprocesses and different color factors.*

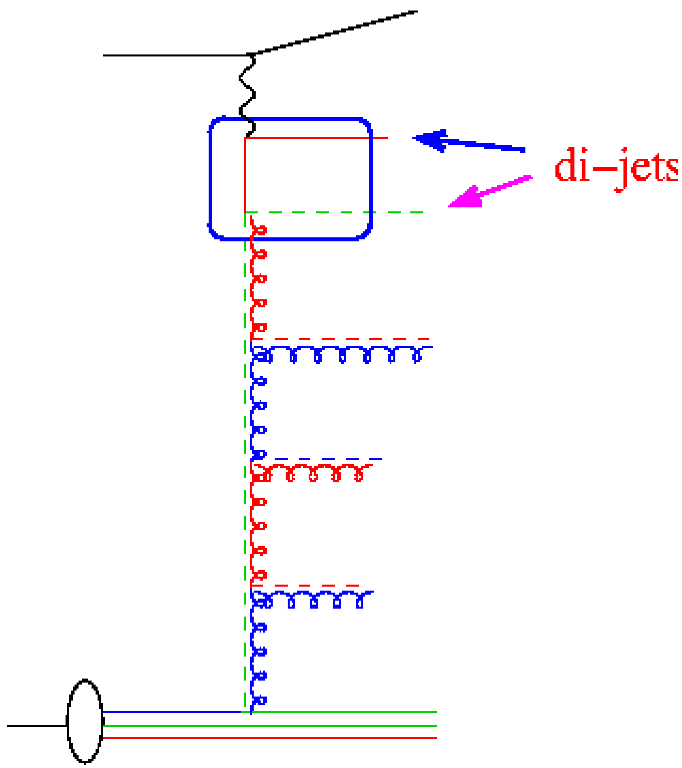
Reduced Scale Dependence in NLO

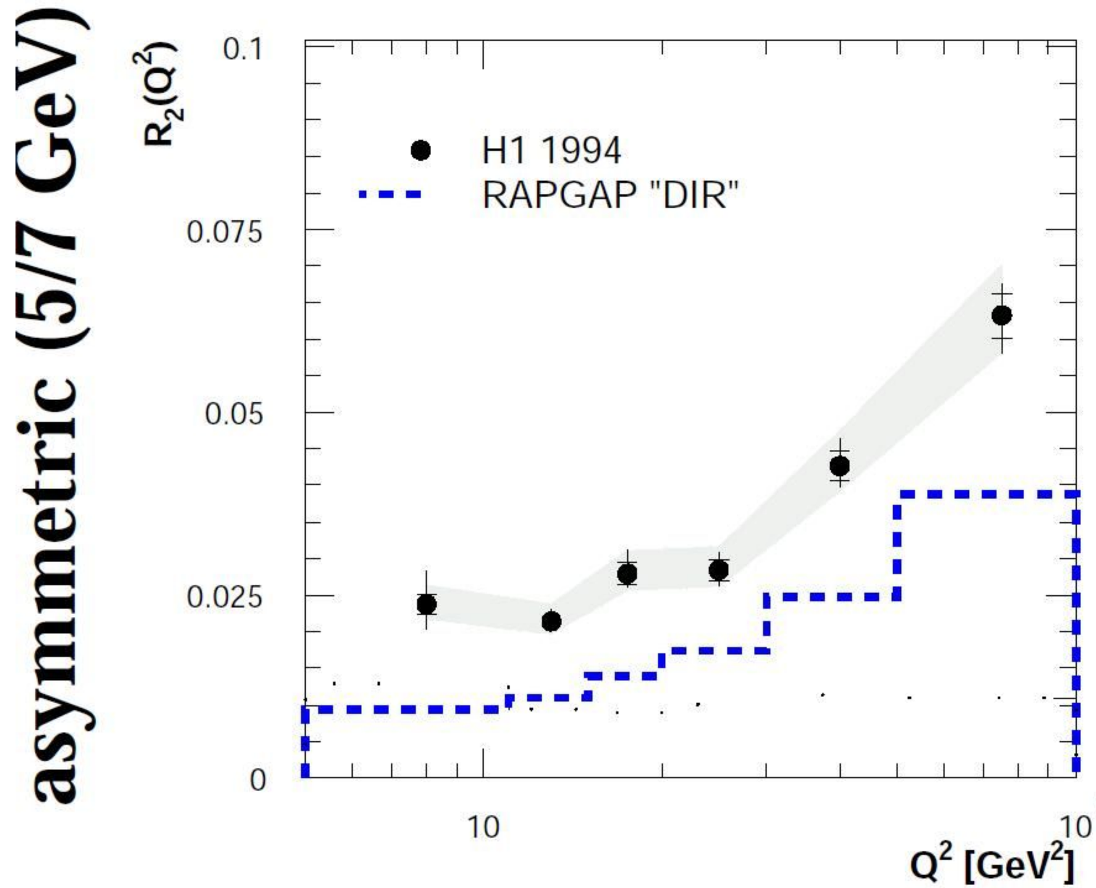
Catani, Seymour hep-ph/9609521

- dependence of the specific choice of the scale for renormalization and factorization shows sensitivity to higher order contributions, which are not included.
- scale is unphysical parameter
- physical observables must be independent of scale
- in NLO scale dependence significantly reduced compared to lowest order



Di-jet rates at LO ?





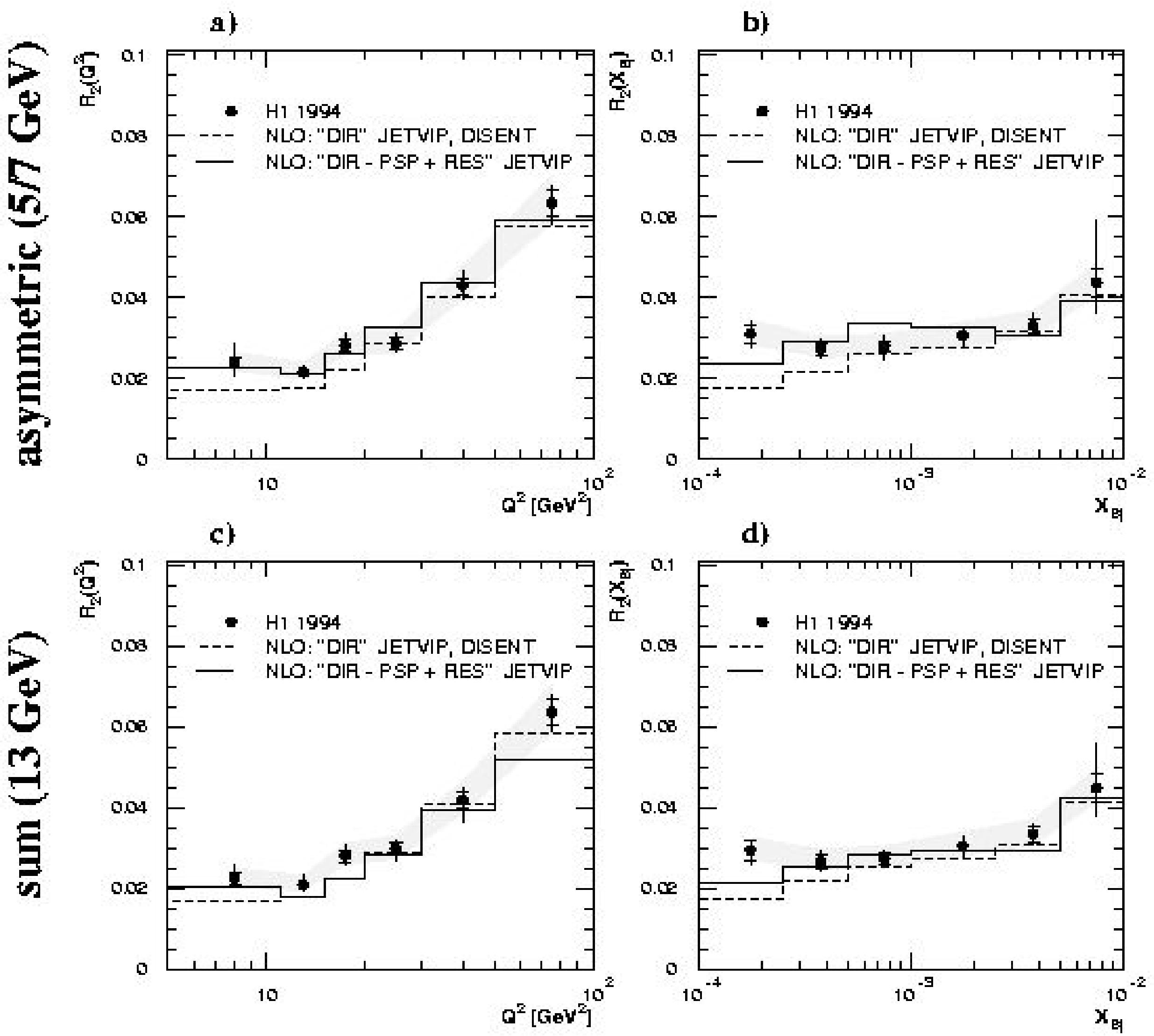
- (2+remnant) jets in DIS for $Q^2 > 5\ GeV^2$, $p_t^{jets} > 5$ GeV
- $\mathcal{O}(\alpha_s)$ processes not enough

➔ need higher order contributions

Diet production at NLO

H1 Collab., C. Adloff et al., Eur. Phys. J. C13 (2000) 415-426

- lowest order NOT enough to describe dijet rates !
- NLO for dijets needed
  - BUT require asymmetric pt to ensure cancellation of real and virtual corrs



Limitations in fixed order NLO calculations

- NEED asymmetric p_t cuts: $p_{t1} \neq p_{t2}$
  for proper cancellation of real and virtual emissions....

➔ loose most of the data... !!!

- improvements by resummations:

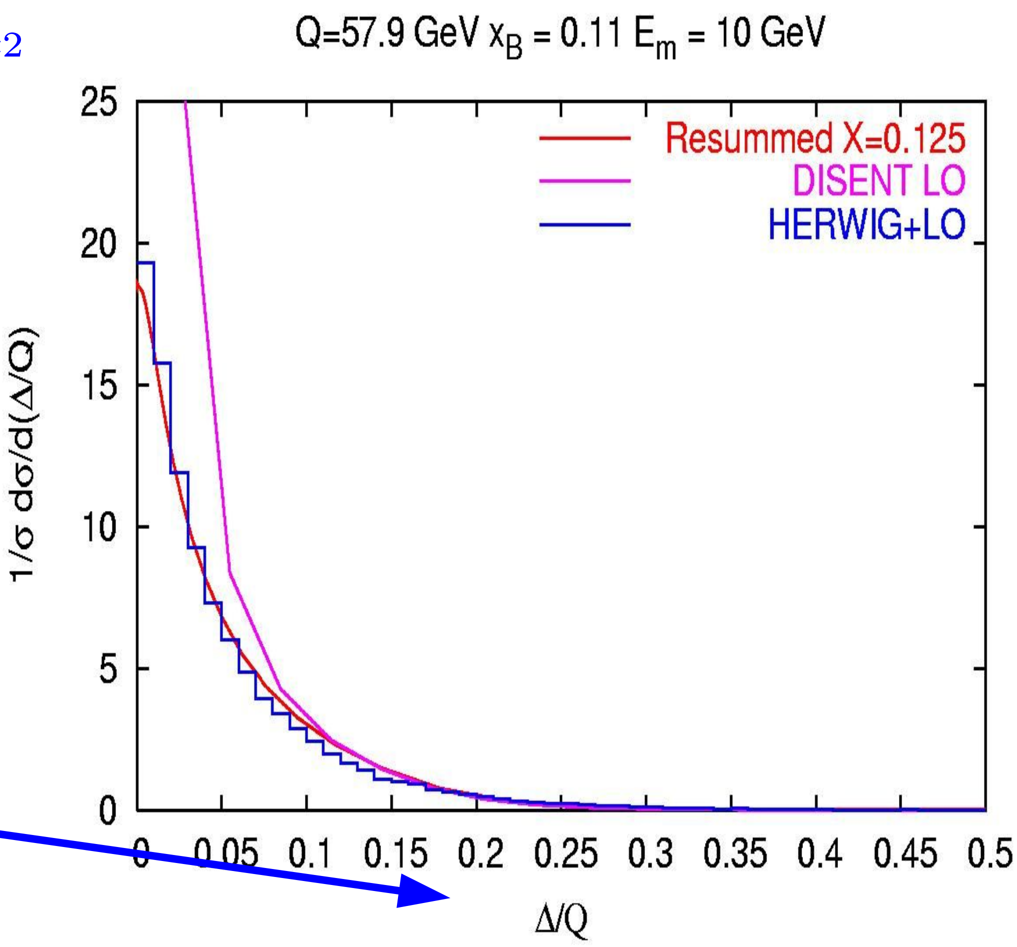
A. Banfi et al hep-ph/0508096

  - soft gluon radiation.... like parton showers... resummed to all orders

  check dijets:

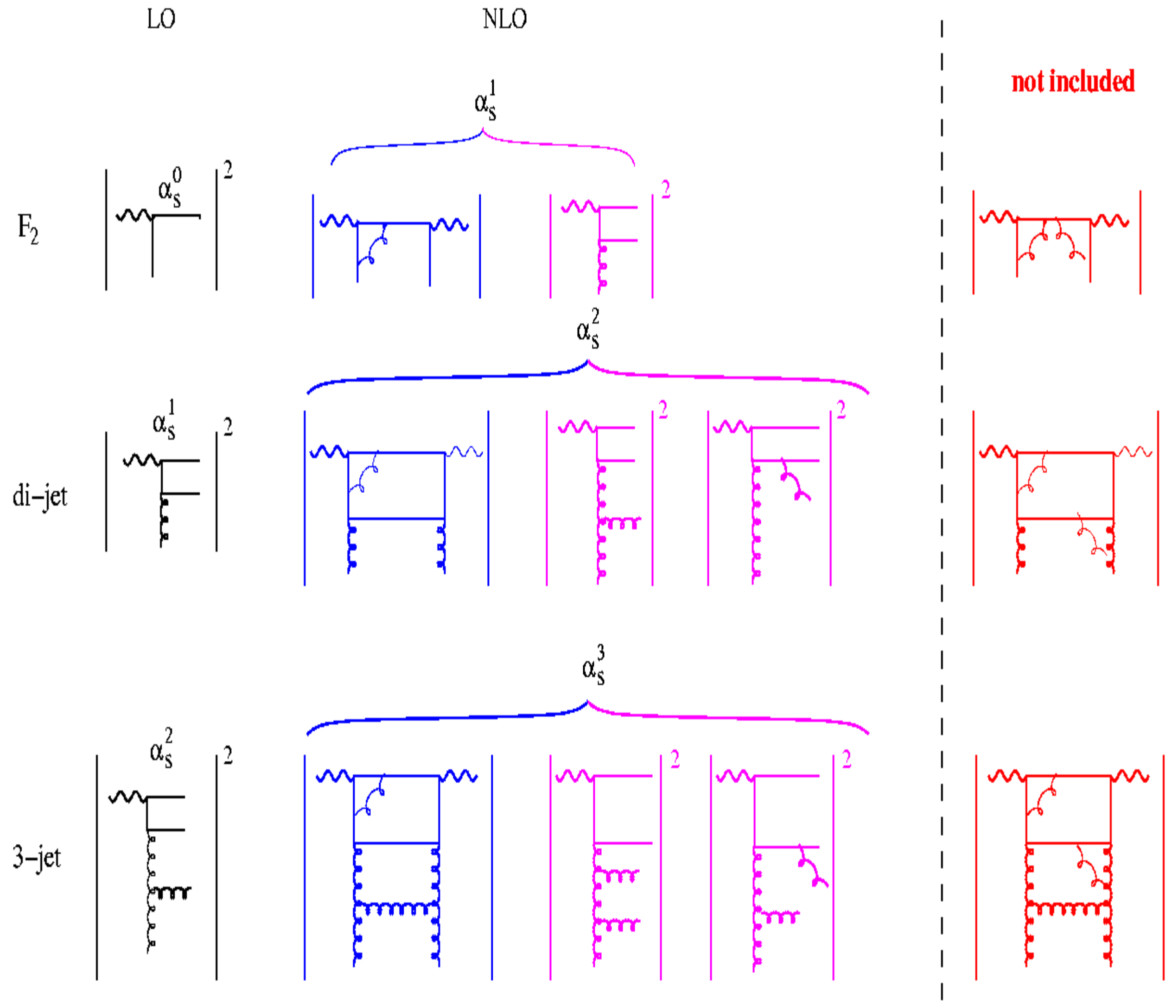
$$\Delta p_t = p_{t1} - p_{t2}$$

  - resummed result at agrees with MC using parton showers...



From LO to NLO ...

- NLO for F_2: $O(\alpha_s)$
- NLO for dijets: $O(\alpha_s{}^2)$
- NLO for 3-jets: $O(\alpha_s{}^3)$



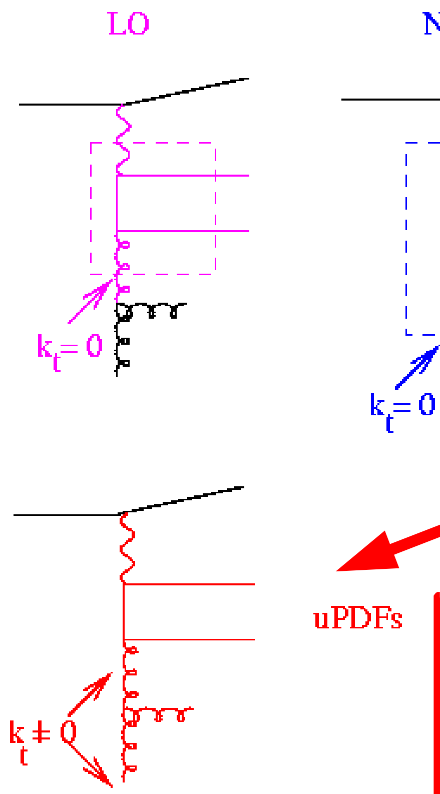
NOTE: NLO for dijets is **NOT** NNLO for F_2

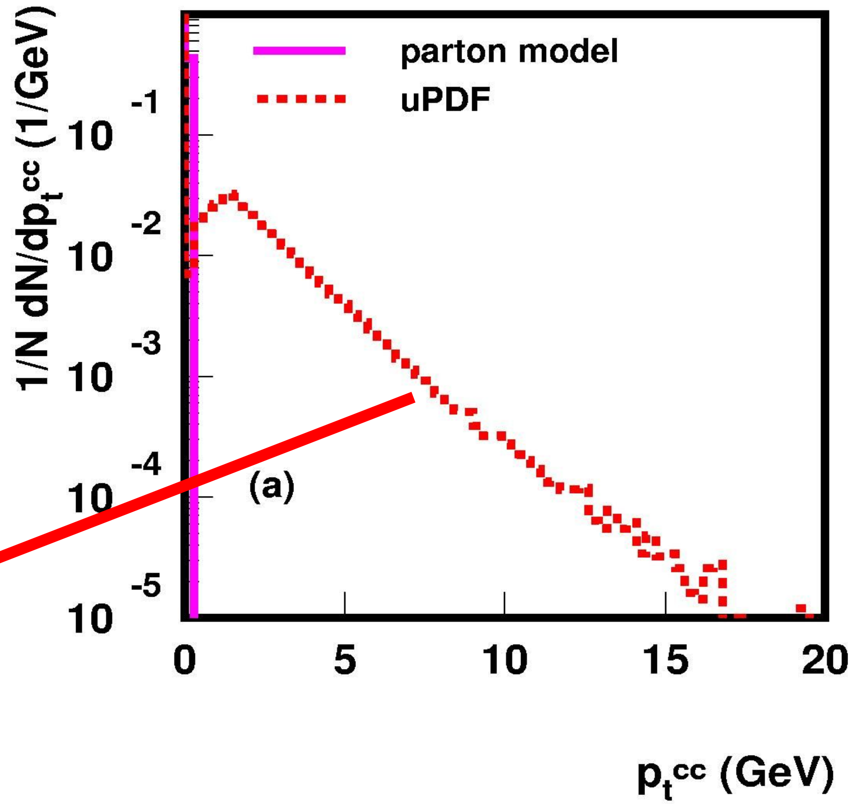
The need for unintegrated PDFs

- using integrated pdfs ignores proper kinematics
- large NLO corr comes from wrong kinematics in LO

Watt, Martin, Ryskin, Eur. Phys. J. C3, 73 (2003)
Watt, Martin, Ryskin, Phys. Rev. D70, 014012 (2004)
Collins, Zu, JHEP 03, 059 (2005)

J. Collins, H. Jung, hep-ph/0508280





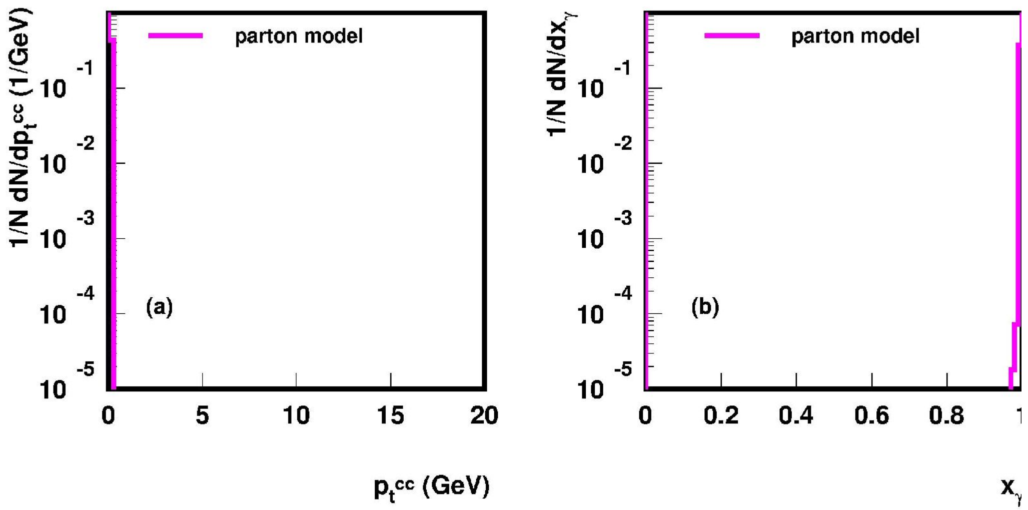
- collinear factorization is wrong if details of final state are investigated
- Need for fully unintegrated PDFs

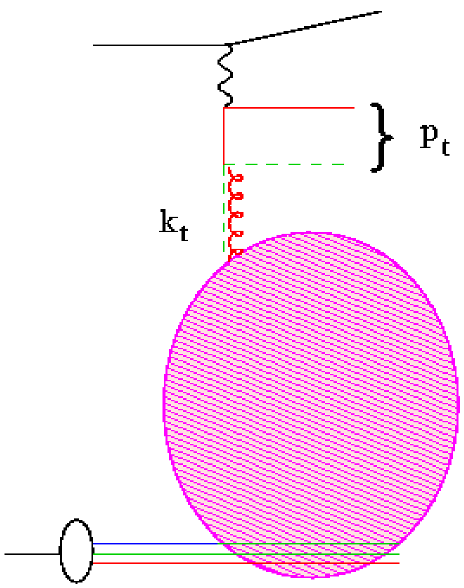
Need for uPDFs

J. Collins, H. Jung, hep-ph/0508280

Define:

- $p_{T\,q\bar{q}}$
- $x_\gamma = \frac{\sum_{i=q,\bar{q}}(E_i - p_{z\,i})}{2yE_e} = \frac{p^-_{q\bar{q}}}{q^-}$
- parton kinematics





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J. Collins, H. Jung, hep-ph/0508280

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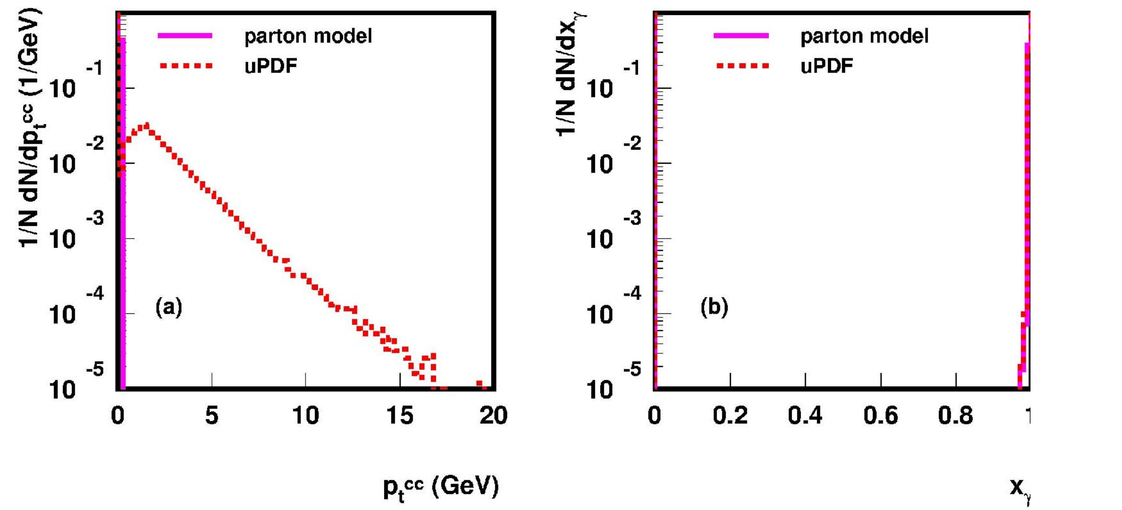
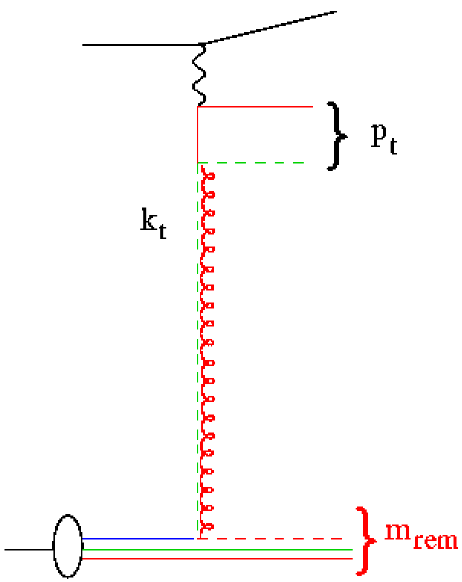
- parton kinematics
- uPDFs



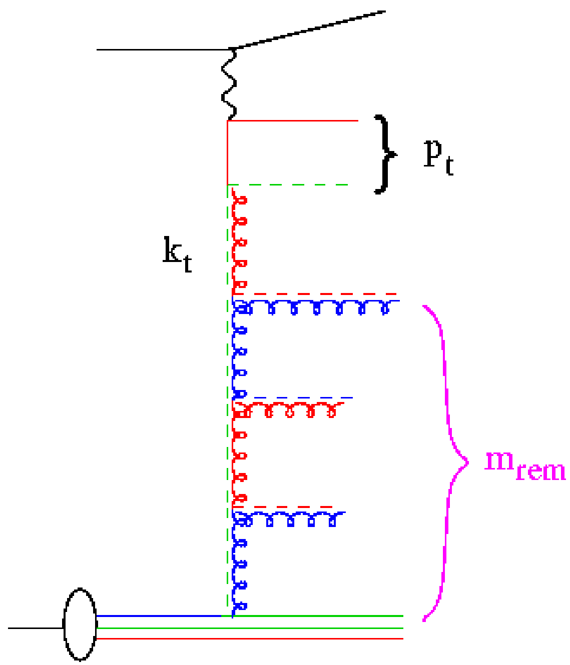


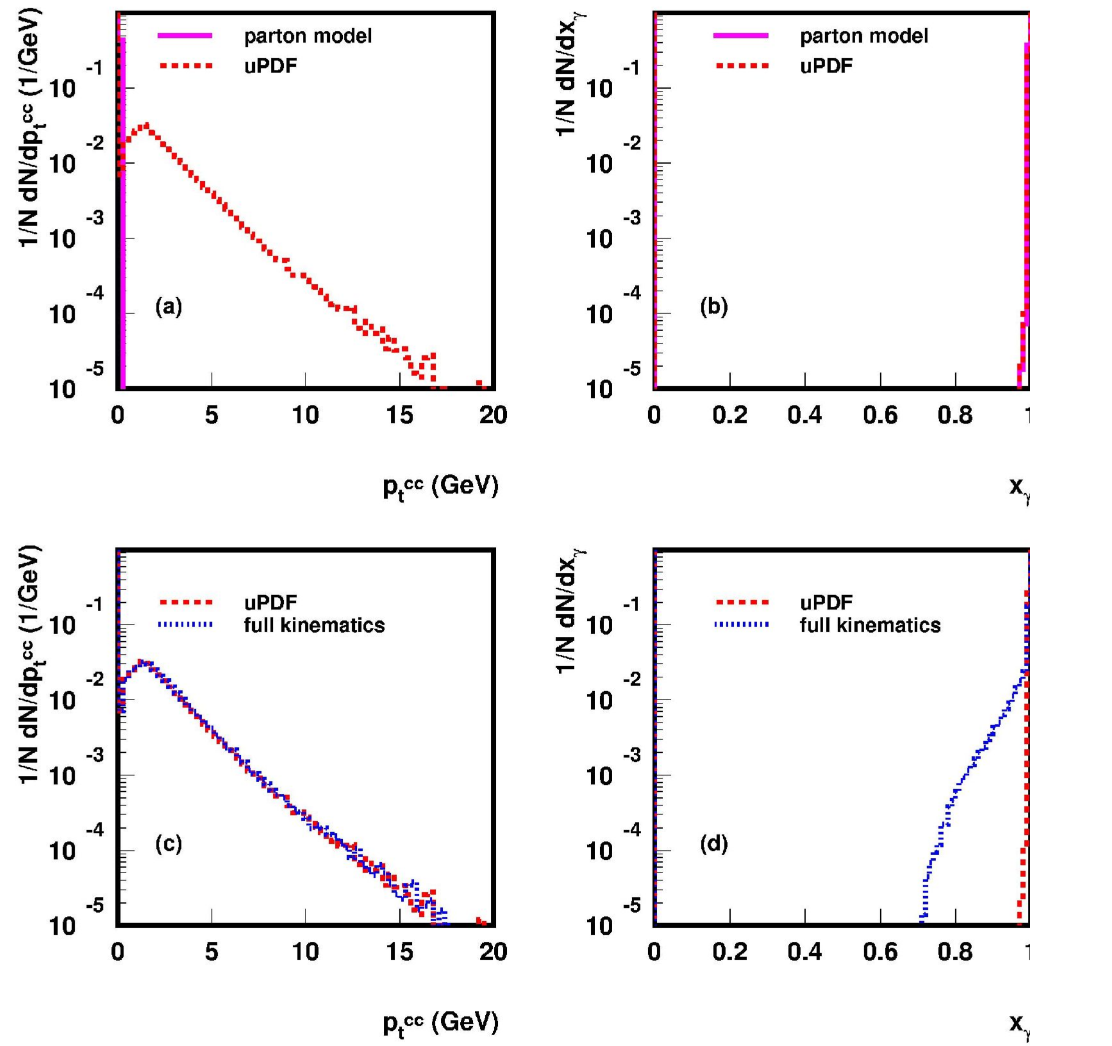
Need for uPDFs

J. Collins, H. Jung, hep-ph/0508280

Define:

- $p_{T\,q\bar{q}}$
- $x_\gamma = \frac{\sum_{i=q,\bar{q}}(E_i - p_{z\ i})}{2yE_e} = \frac{p^-_{q\bar{q}}}{q^-}$
- parton kinematics
- uPDFs
- full kinematics

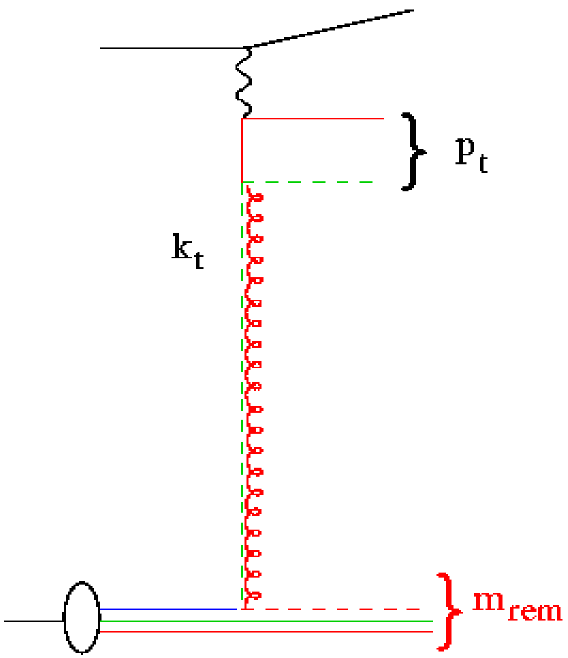


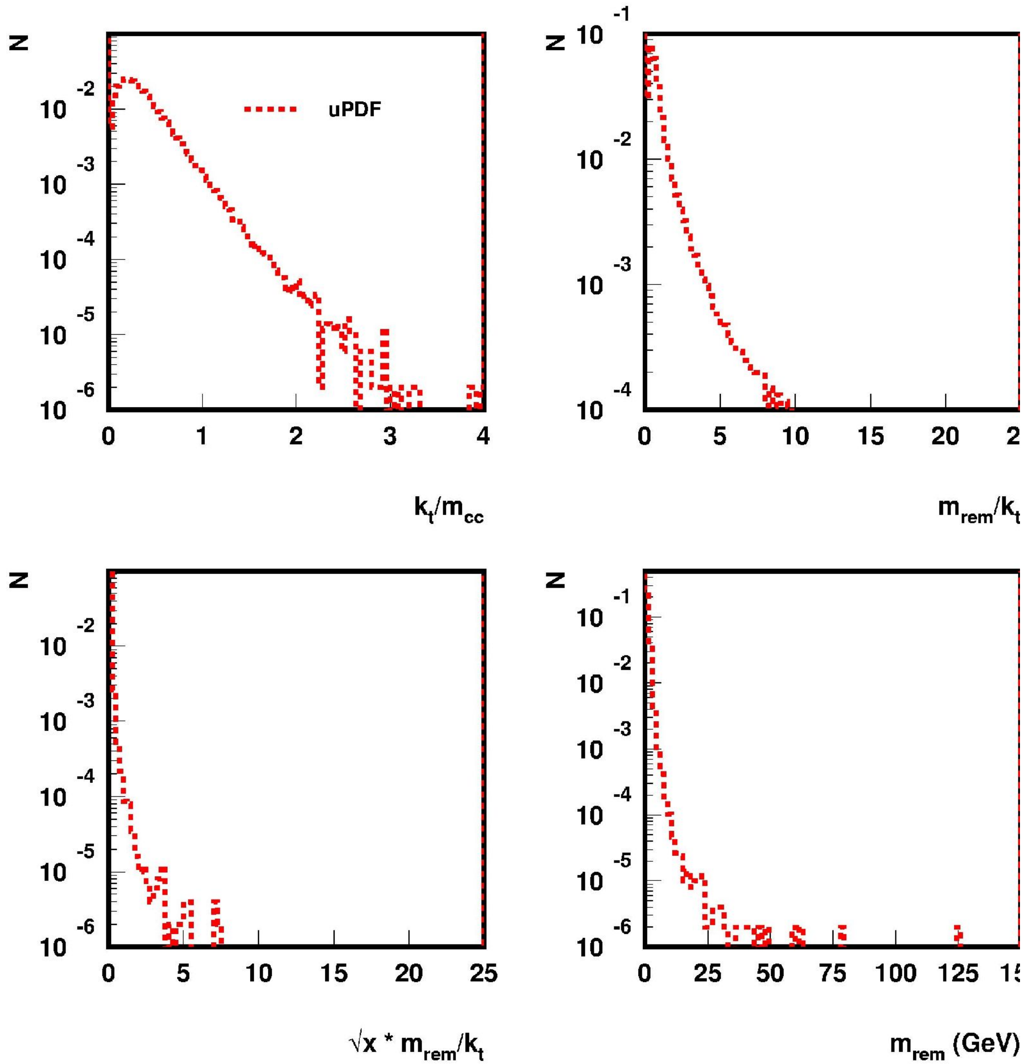


Need for double uPDFs

J. Collins, H. Jung, hep-ph/0508280

$$k^2 = -\frac{k_t^2}{1-x}$$



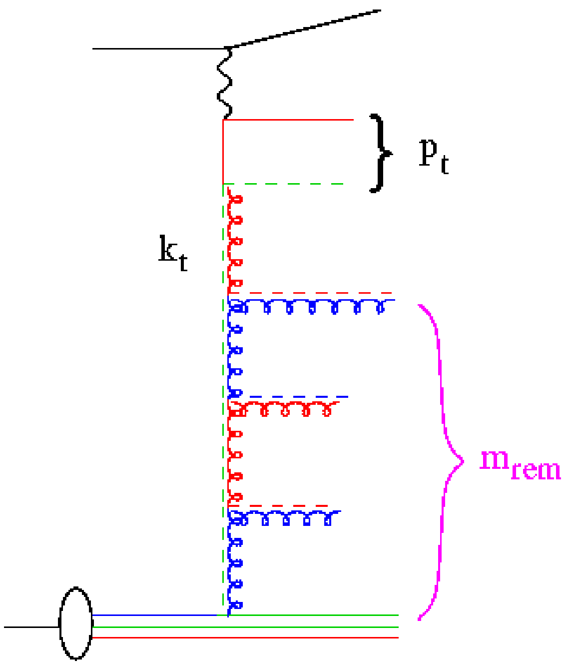


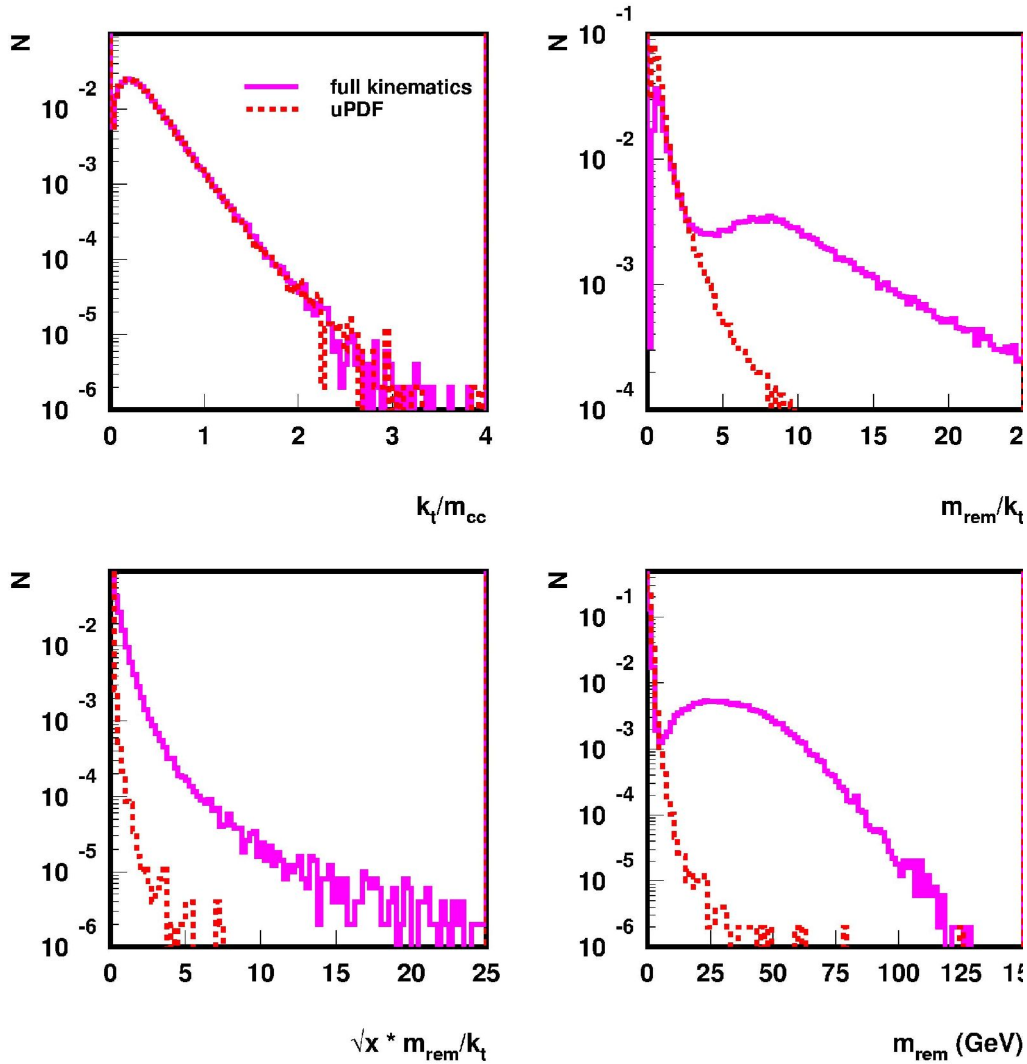
Need for double uPDFs

J. Collins, H. Jung, hep-ph/0508280

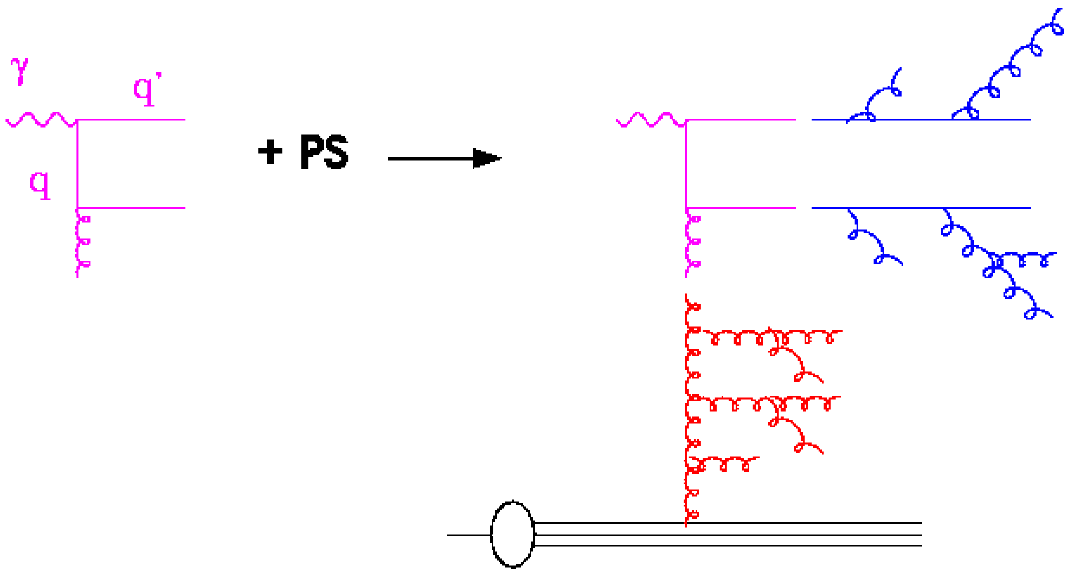
$$k^2 = -\frac{k_t^2}{1-x}$$

$$k^2 = -\frac{k_t^2}{1-x}\left(1 + x\frac{m_{rem}^2}{k_t^2}\right)$$





Explicit parton evolution: parton showers



- use LO matrix elements
  - for light quarks, cutoffs are needed
- apply initial and final state parton showers (**PS**)
  - matching of cutoff in ME with parton showers
- apply synchronization
- obtain cross sections fully differential in any observable
- **BUT:**
  - only in LO (attempts to include NLO: Collins et al, MC@NLO, etc)

DGLAP evolution again and again...

- differential form: $$t\frac{\partial}{\partial t} f(x,t) = \int \frac{dz}{z}\frac{\alpha_s}{2\pi} P_+(z)\, f\left(\frac{x}{z},t\right)$$

- differential form using f/Δ_s with

$$\Delta_s(t) = \exp\left(-\int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}\tilde{P}_2(z)\right)$$ with $$\tilde{P}_2 \sim \frac{1}{1-z}$$

$$t\frac{\partial}{\partial t}\frac{f(x,t)}{\Delta_s(t)} = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{\tilde{P}(z)}{\Delta_s(t)} f\left(\frac{x}{z},t\right)$$

- integral form

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'}\cdot\frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z},t'\right)$$

no – branching probability from t_0 to t

DGLAP for parton showers

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via explicit iteration:

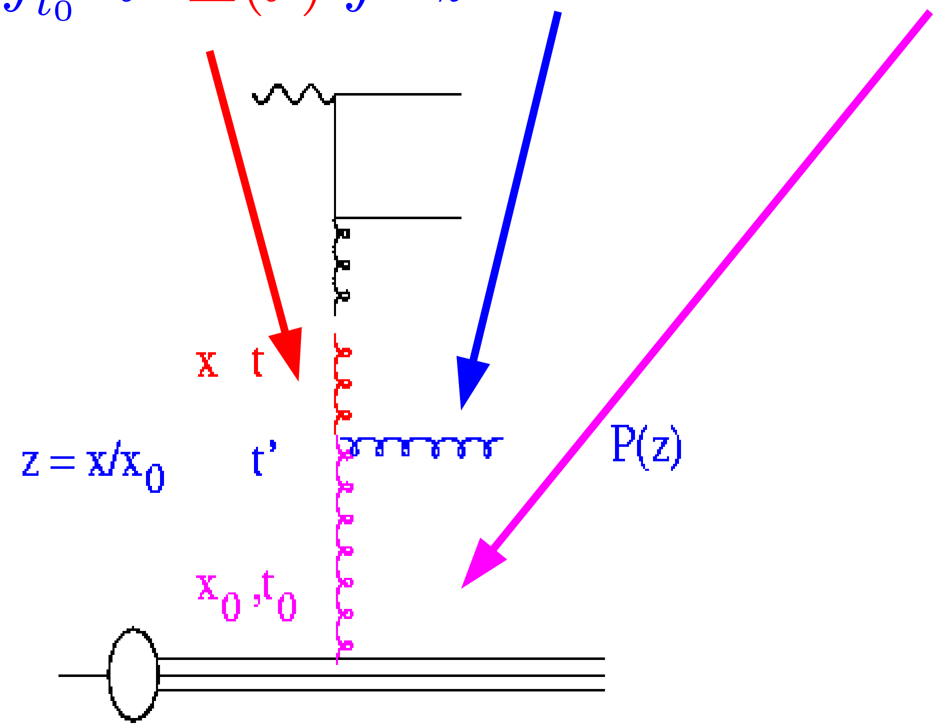
from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching

$$f_0(x,t) = f(x,t_0)\Delta(t)$$

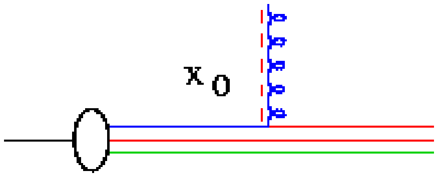
$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0)\Delta(t')$$



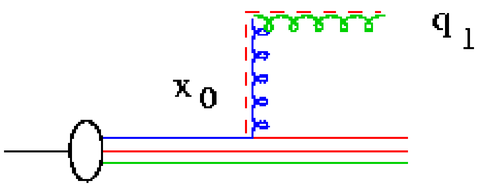
Parton showers for the initial state

spacelike (Q<0) parton shower evolution

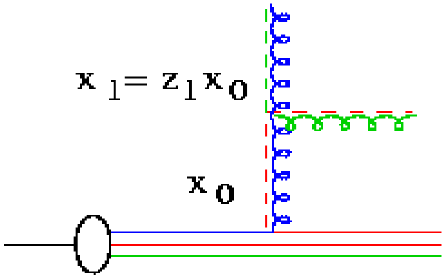
- starting from hadron (fwd evolution)
  or from hard scattering (bwd evolution)



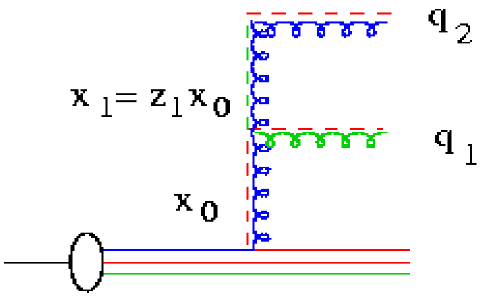
- select q_1 from Sudakov form factor



- select z_1 from splitting function



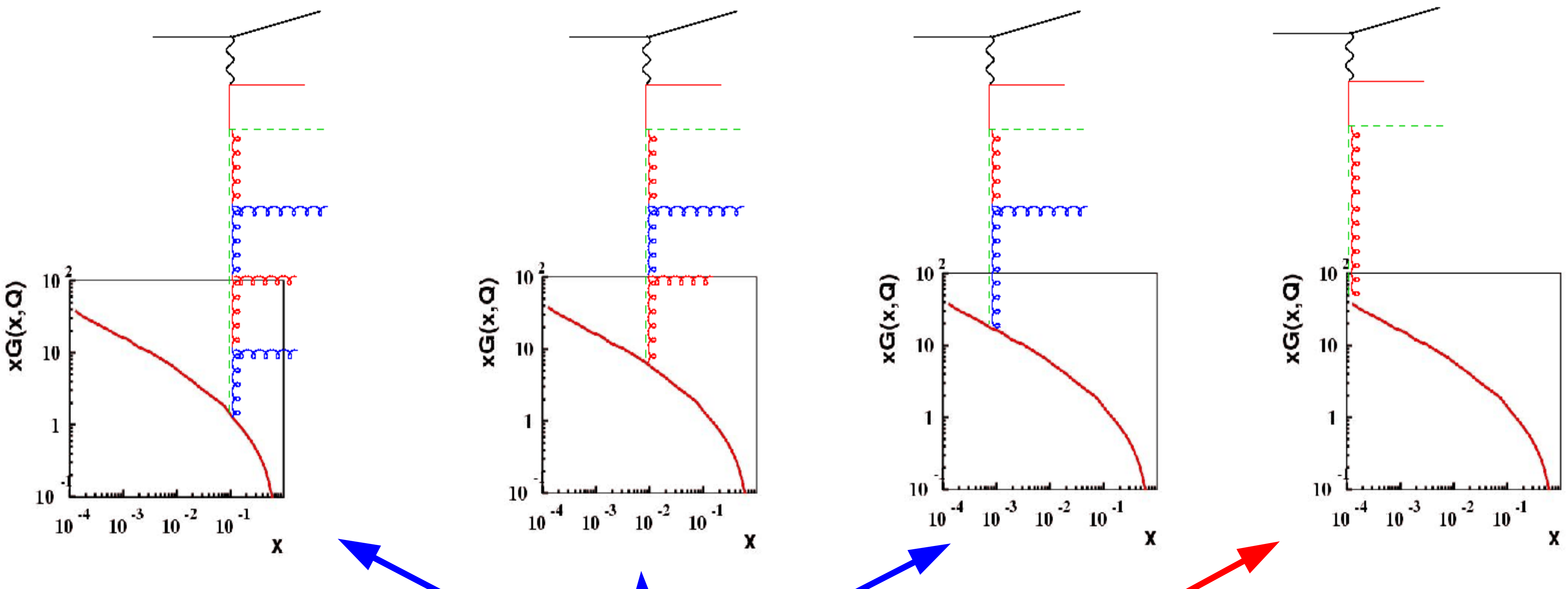
- select q_2 from Sudakov form factor



- select z_2 from splitting function
- stop evolution if $q_2 > Q_{hard}$

Parton showers to solve DGLAP evolution

- for fixed x and Q^2 chains with different branchings contribute
- iterative procedure, spacelike parton showering

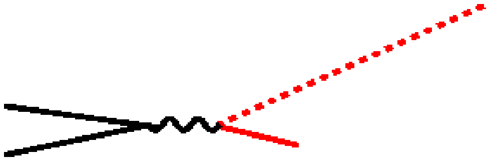


$$f(x,t) = \sum_{k=1}^{\infty} f_k(x_k, t_k) + f_0(x, t_0)\Delta_s(t)$$

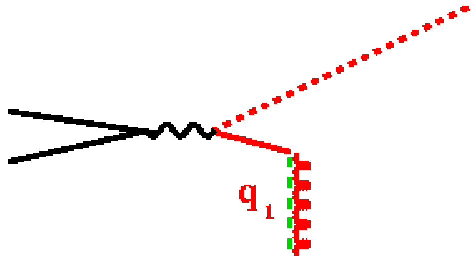
Parton showers for the final state

timelike (Q>0) parton shower evolution

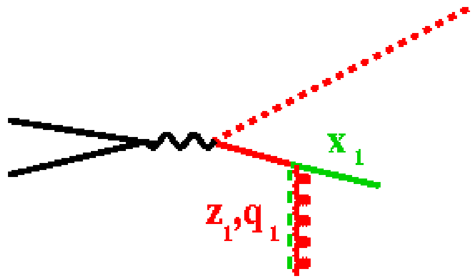
- starting with hard scattering



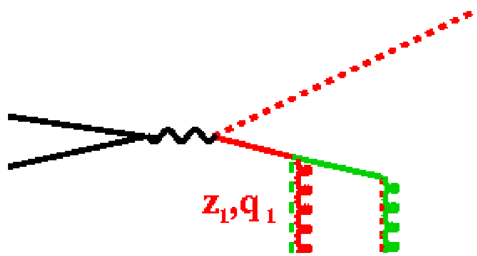
- select q_1 from Sudakov form factor



- select z_1 from splitting function



- select q_2 from Sudakov form factor



- select z_2 from splitting function
- stop evolution if $q_2<q_0$