ICE-EM / AMSI Summer School 2007
List of Courses

At the discretion of their home institution, honours students may be able to take courses for credit towards their degree. Contact your Head of Department or Honours Coordinator for guidance on this matter.

Measure Theory

Duration Four Weeks

Lecturer Marty Ross (Melbourne)

Consultation Hours: Tuesdays 16–17h Carslaw Room 821

Assessment 50% problems assigned in lectures and 50% take home exam.

Assumed Knowledge We will assume familiarity with the fundamental concepts of analysis in Euclidean Space (infimum and supremum, open and closed sets, continuity, completeness and compactness, countability). Some corresponding familiarity with these notions in metric spaces would be helpful but will not be assumed; familiarity with these notions in topological spaces would be just peachy.

Background Material Lecture notes summarising the relevant background on sets and real analysis (PDF) are available. Some (but definitely not all) of this material will be reviewed along the way, particularly the material on metric spaces and topological spaces. Before the summer school begins, you should definitely take a good look at the background notes and, if need be, browse through a real analysis text or two.

Course Outline Measure theory is the modern theory of integration, the method of assigning a "size" to subsets of a universal set. It is more general, more powerful and more beautiful (though also more technical) than the classical theory of Riemann integration. The course will be a reasonably standard introduction to measure theory, with some emphasis upon geometric aspects. We will cover most (but definitely not all) of the topics listed below, subject to time and taste:

- General Measure Theory (Outer measure; Measurable sets; Borel and Radon measures; the Caratheodory criterion for Borel measures)
- Special Measures on Euclidean Space (Lebesgue measure; Hausdorff measure; the Vitali Covering Theorem; Hausdorff dimension)
- Integration (Measurable functions; integration and convergence theorems; the Area Formula; iterated integrals and Fubini's Theorem)
- Functional Analysis (Measures as linear functionals; L_p spaces; the Riesz Representation Theorem)
- Further Topics (Differentiation of measures; the Besicovitch Covering Theorem; the Generalised Fundamental Theorem of Calculus; the Co-Area Formula)

Differential Geometry

Duration Four Weeks

Lecturer Tanya Schmah (Macquarie University)

Consultation Hours: Tuesday 12-13, 14-15 in Carslaw Room 609. (These are times when I promise to be there and available. Please feel free to make an appointment, or just drop by, at other times.

Assessment 80% final exam (Friday 9 Feb 9–11 in Carslaw Room 275), 20% assignments.

Assumed Knowledge A good grounding in multivariable calculus, linear algebra and introductory real analysis, as well as some familiarity with ODEs and groups. Some experience of classical mechanics is desirable but not necessary.

Resources There is an Assignment (PDF) part of which you are expected to hand in by Wednesday in the first week (not part of the assessment). See http://www.maths.mq.edu.au/~schmah/ice07/ for more resources.

Course Outline This is an introduction to Riemannian and symplectic geometry on manifolds, with some applications to mechanics. We begin with the geometry of parametrised surfaces, including: Gaussian curvature; geodesics; isometries and area-preserving maps; and gradient and skew-gradient vector fields. The second part of the course is about manifolds, including: vector fields, tensors and forms; Lie derivatives; Frobenius' theorem; orientation; and Lie groups. In the latter third of the course, we formally introduce Riemannian geometry, symplectic geometry and Hamiltonian mechanics. Time permitting, the course will conclude with an introduction to Lie group symmetries in mechanics.

Lie Algebras

Duration Four Weeks

Lecturer Anthony Henderson (University of Sydney)

Consultation Hours Tuesdays 13–14h Carslaw Room 805.

Assessment 80% Final exam (Thursday 8 Feb 9–11 in Carslaw Room 275), 20% assignments.

Resources Lecture notes available for purchase at KopyStop Shop 3 / 55 Mountain St Broadway (see the Google Map for the location). An information sheet (PDF) is available.

Course Outline This course will be an introduction to the classical theory of complex Lie algebras and their representations. As well as underpinning modern representation theory, and relating to other fields such as differential geometry and mathematical physics, this area is accessible and beautiful in its own right. Some familiarity with linear algebra and the basic algebraic notions of groups and modules will be assumed, but the lecture notes will include appendices reviewing what is needed of these topics.

Cryptography

Duration Four Weeks

Lecturer David Kohel (University of Sydney)

Assessment 80% Final exam (Friday 9 Feb 14–16h in Carslaw Room 351), 20% assignments.

Consultation Hours: Thursdays 14–15h Carslaw Room 625.

Assumed Knowledge We will assume a basic background of abstract algebra: (primarily abelian) groups, rings, and fields. Students with some background on elementary number theory, or modular arithmetic (the ring Z/NZ) should be able to pick up the relevant concepts along the way.

Background Material Any standard textbook on abstract algebra (e.g. Fraleigh's "A first course in abstract algebra", Herstein's "Abstract algebra", or chapter one of Lidl and Niederreiter's "Finite Fields") would be useful, but relevant material will be summarised with appendices to the lectures notes. The popular book "The Codebook" by Simon Singh is recommended as entertaining background reading on the subject.

Resources Lecture notes and other resources are posted at http://echidna.maths.usyd.edu.au/~kohel/tch/Crypto/.

Course Outline We begin with an overview of cryptography, including the fundamental concepts of encryption, material on classical ciphers and their cryptanalysis, and distinction between symmetric key and asymmetric (or public) key cryptography. Material on symmetric key cryptography will cover standard block cipher constructions, together with their modes of use, followed by stream ciphers. For the subject of public key cryptography, we recall the construction of the quotient rings Z/NZ and introduce finite fields of any prime power. The main cryptosystems RSA and ElGamal will be introduced, together with the main algorithms for their construction (finding large primality and proving primality) and their cryptanalysis (factoring and discrete logarithm algorithms). We conclude with more "exotic" generalisations of ElGamal cryptosystems to groups of elliptic curves.

Throughout we emphasize an algorithmic approach, and demonstrate the principles with computer experiments and exercises. This will allow us to provide nontrivial examples and discuss algorithmic approaches to construction of large prime numbers, factorization, and "discrete logarithms".

Wavelets and Computation

Duration Four Weeks

Lecturer Markus Hegland (Australian National University)

Consultation Hours By appointment, contact the lecturer.

Assessment Assignments, project and exam, roughly half for the project and half for the assignments and exam. For more detail see http://wwwmaths.anu.edu.au/~hegland/amsi07/.

Assumed Knowledge The course should be accessible and of interest to students doing mathematics, science, computer science and engineering who have done one to two years of advanced calculus and linear algebra. Familiarity with concepts like continuity, differentiability, convergence, and Taylor series is assumed both for one and multidimensional functions. From linear algebra we will need vector spaces and linear mappings, orthogonality and eigenvalues. Some familiarity with Lebesgue measures, Hilbert spaces and Fourier series and transforms is an advantage but results from these areas will be reviewed during the course.

Background Material Revise the material as covered in a good analysis book like W. Rudin: *Principles of Mathematical Analysis* or a calculus book like R. A. Adams: *Calculus – A Complete Course*.

Most of the material covered in the course can be found in: S. Mallat, *A Wavelet Tour of Signal Processing*.

There is a vast amount of literature on wavelets, a reference from the early days of wavelets (15 years ago) is I. Daubechies: *Ten Lectures on Wavelets*. This book is for the mathematically inclined reader, a more computational reader may enjoy: *Wavelets and Filter Banks* by Gilbert Strang and Truong Nguyen. A very readable introduction by one of the pioneers of wavelet research in the 80s is: *Wavelets: Algorithms & Applications* by Yves Meyer. Other references will be provided throughout the course.

Resources Lecture notes and other supporting material will be posted at http://wwwmaths.anu.edu.au/~hegland/amsi07/.

Course Outline The course provides an introduction to wavelets, with a particular focus on computational aspects including approximation and numerical linear algebra. During the course, the mathematical foundations and concepts required for time-frequency analysis and multi-resolution will be extensively discussed.

The course has three main themes: In the first weeks, we discuss Fourier analysis, from convolutions to the Heisenberg uncertainty. This is followed by a discussion of discrete Fourier transforms, windowed transforms up to the fast Fourier transform (FFT). The second theme are the actual wavelets. After an introduction to time/frequency analysis we will discuss connections between regularity and the size of the wavelet transform, vanishing moments and multi-resolution. The third theme is computational. Here we cover particular wavelet basis followed by an introduction to approximation theory using Fourier analysis and Sobolev spaces for the linear case and Besov spaces for the nonlinear case. If there is time at the end we will review applications of wavelets to machine learning, image processing and compression. Furthermore, we will consider the implementation of the fast wavelet transform and applications in high-dimensional approximation.

The course consists of lectures and tutorials where the students will be able to actively explore either given topics or questions they are interested in which relate to the course. The ratio of lectures to tutorials is around 3:1 but can vary depending on the interests and backgrounds of the participants.

Time Series Analysis

Duration Four Weeks

Lecturer Niels Wessel (University of Potsdam, Germany) and Hagen Malberg (Forschungszentrum Karlsruhe, Germany) Tutorial preparation: Norbert Marwan

Consultation Hours Wednesdays 11–12 Carslaw Room 628.

Assessment 80% Final exam (Friday 9 Feb 14–16h in Carslaw Room 350), 20% assignments.

Course Outline Time series analysis, i.e. in time sequentially ordered observations, is of arising importance in different scientific fields, particularly in medicine. The course gives an introduction into the specific features and application of biomedical time series analysis in science and praxis. The course contains the presentation of the biological background. Physiological regulatory systems, which are relevant in clinical diagnosis, are presented.

Beside neuronal and internal processes, the main focus is set on the cardiopulmonal system, the autonomic cardiovascular regulation. How to measure physiological biosignals and how to extract artifact- and noiseless time series from the biosignals, these tasks are necessary for analysing the physiological processes, as a precondition. The course presents the number of classical and new mathematical approaches for time series analysis: stationarity analysis, time and frequency domain methods, methods of non-linear dynamics, analysis of seldom physiological events (i.e. arrhythmias) and coupling phenomena. Furthermore, knowledge-based methods, i.e. fuzzy and neuronal approaches are introduced.

The course also presents the application of mathematical methods for clinical diagnosis and risk stratification. How to perform a clinical study and how to generate new physiological knowledge from the calculated parameters are introduced. Additionally to the course theoretical and practical exercises are offered, where also real data sets from medicine will be analysed. An introduction into the used software will be given.

Stochastic Calculus

Duration Two Weeks, Period 1 (followed by Financial Mathematics)

Lecturer John van der Hoek (University of Adelaide)

Consultation Hour Week 1: Tuesday 17–18h, Week 2: Wednesday 15–16h in Carslaw Room 633.

Assessment take home exam to be posted to Dr John van der Hoek, School of Mathematical Sciences, The University of Adelaide, SA 5005 by date TBA.

Course Outline

- Probability preliminaries [including sigma algebras, filtrations and conditional expectations, martingales]
- Brownian motion and its characterrizations
- Ito integrals and related integrals
- Ito's Lemma
- Stochastic Differential Equations
- Partial Differential Equations and Feynman-Kac formula
- Changes of Probability and Girsanov's Theorem
- Martingale representation theorems

Financial Mathematics

Duration Two Weeks, Period 2 (following Stochastic Calculus)

Lecturer David Colwell (School of Banking & Finance, University of New South Wales)

Consultation Hours Consult lecturer

Assessment Final exam in last lecture (Friday 9 January, 4–6pm in Carslaw Room 275)

Assumed Knowledge Stochastic Calculus

Course Outline This module will consider the pricing and hedging of financial derivatives from a mathematical viewpoint. In particular, the topics covered will include:

- Mathematical model of the underlying financial market.
- Absence of arbitrage and market completeness.
- Probabilistic approach to pricing and hedging contingent claims under the Black-Scholes framework.
- Pricing and hedging contingent claims under stochastic volatility, and in incomplete markets in general.
- Traditional short rate models.
- Heath, Jarrow, and Morton framework for term structure modelling.
- Term structure models with finite dimensional realisations.
- Brace, Gatarek, and Musiela model.

Geophysical Fluid Dynamics

Duration Two Weeks, Period 1

Lecturer Marcel Oliver (International University Bremen, Germany)

Consultation Hours Thursdays 11-12h Carslaw Room 635 (Weeks 1 & 2 only).

Assessment 5 short assignments.

Assumed Knowledge A first course on PDEs or equivalent exposure, a rigorous course in Analysis as well as vector calculus. The following will help, but are not essential prerequisites: functional analysis and some exposure to fluid mechanics.

Course Outline This course is an introduction to mathematical problems in geophysical fluid dynamics. The course is structured in three parts:

Part I: The equations of geophysical fluid dynamics

- The rotating Euler equations, the Boussinesq approximation, hydrostatic balance
- Energy and vorticity; vorticity-streamfunction formulation in two-dimensional flows
- Special solutions; linear and nonlinear stability
- Shallow water and nearly-geostrophic limit equations

Part II: The quasi-geostrophic equations as a singular partial differential equations limit.

- Review of function spaces and basic facts from Functional Analysis
- Well-posedness of shallow water and quasi-geostrophic equations
- Convergence theory for balanced initial data
- Unbalanced data: Embid-Majda theory
- Unbalanced data: Babin-Mahalov-Nikolaenko theory

Part III: Further topics (if sufficient time and interest)

- Nonlinear dispersive waves
- Semi-geostrophic limits
- Variational methods

References The lectures are loosely based on the book by A. Majda, *Introduction to PDEs and waves for the atmosphere and ocean*, American Mathematical Society, 2003, supplemented by additional reading and original research papers.

Dynamical systems

Duration Two Weeks, Period 1

Lecturer Arno Berger (University of Canterbury, New Zealand)

Consultation Hours Thursdays 12–13h Carslaw Room 526.

Assessment Two assignments, one each week.

Assumed Knowledge A good working knowledge in introductory real analysis and linear algebra is essential. Some prior exposure to (ordinary) differential equations as well as to fundamental concepts (such as e.g. convergence, continuity, compactness, completeness, connectedness) in the context of metric spaces would be beneficial.

Course Outline This course provides a concise introduction to dynamics, with an emphasis on geometric and topological aspects. Guided by illustrative examples throughout, we will study how apparently simple systems can exhibit complex and unpredictable ("chaotic") long-time behaviour, and we will develop the mathematical terminology and tools necessary to describe and quantify this complexity in various ways. Topics discussed (with varying level of detail) include:

- stability, instability, bifurcations;
- signatures of chaos: horseshoes, symbolic dynamics, entropy;
- equivalence of systems, classification, robustness;
- topological dynamics: transience, recurrence, expansivity.

Depending on time and interest, we may also have a first look at some more advanced topic, e.g. Conley index, multiple recurrence, and Furstenberg's diophantine theorem.

References Highly readable introductions to dynamics are Devaney: *An Introduction to Chaotic Dynamical Systems* and Hasselblatt & Katok: *A First Course in Dynamics*; the course will considerably overlap with these textbooks. Most of the material covered (and much more) can be found in the more advanced texts Brown: *Ergodic Theory and Topological Dynamics*, Irwin: *Smooth Dynamical Systems*, and Katok & Hasselblatt: *Introduction to the Modern Theory of Dynamical Systems*.